#### (Brain) connectivity

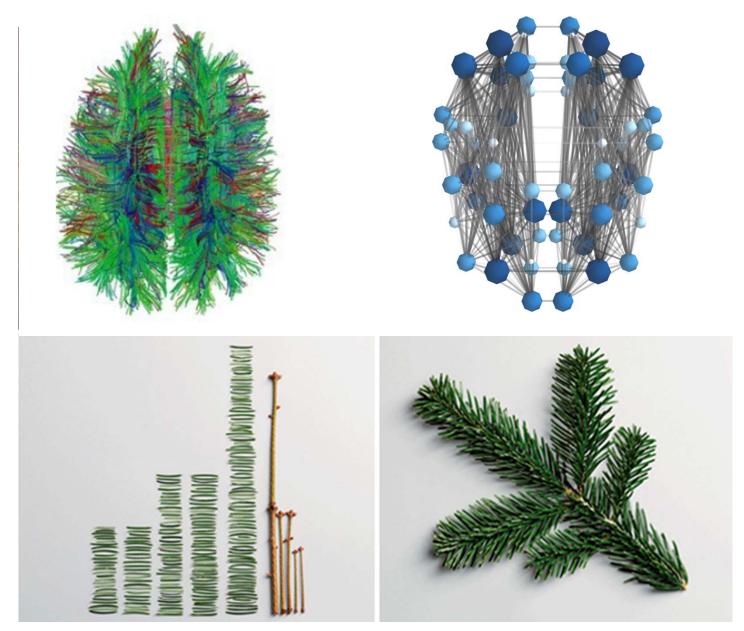
A sort of introduction





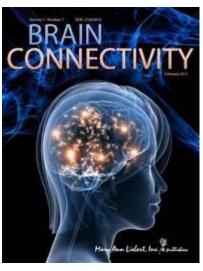


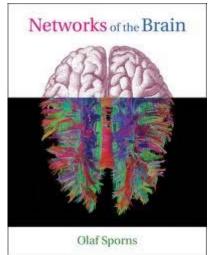
Ursus Wehrli "Kunst aufräumen"

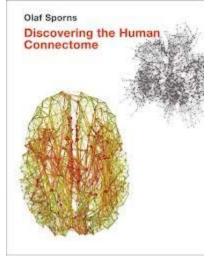


Ursus Wehrli "Kunst aufräumen"









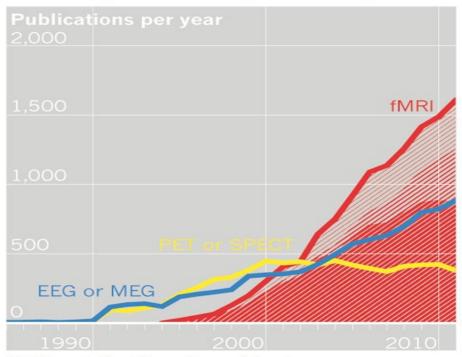






#### THE RISE OF MRRI

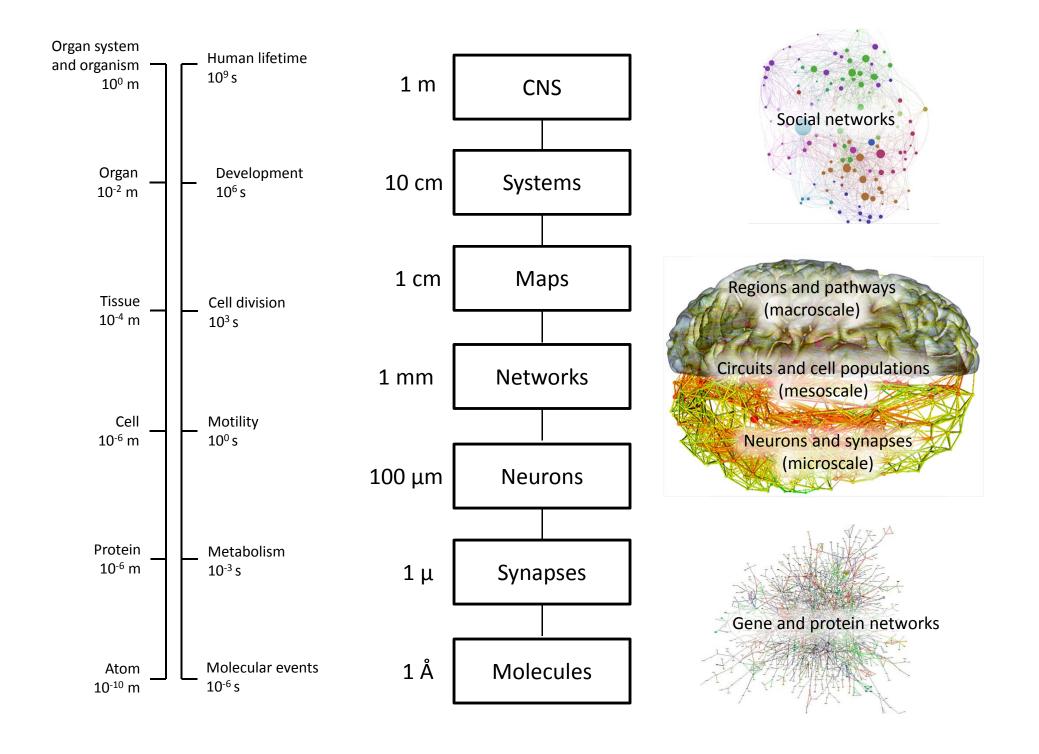
Use of fMRI has rocketed, and now more studies are looking at connectivity between regions.



#### fMRI publications by subject:

Activation Connectivity Milly Other

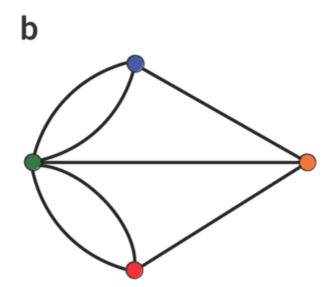
fMRI, functional magnetic resonance imaging; PET, positron emission tomography; SPECT, single-photon emission computed tomography; EEG, electroencephalography; MEG; magnetoencephalography Data from ISI Web of Knowledge.



### Key characteristic 1: Networks as pathways

a





### Key characteristic 2: Networks as an expression of collective dynamics

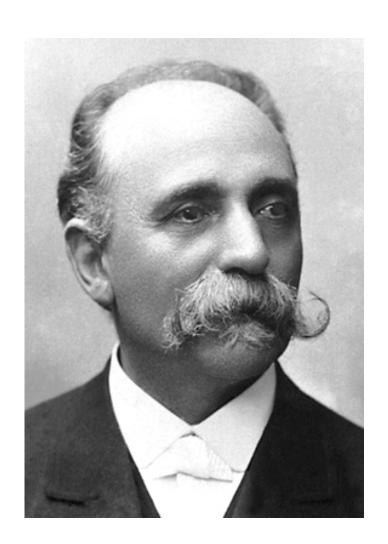


## Adolphe Quetelet (Gent, 1796-1874)

When we consider a big number of individuals, social dynamics are ruled by collective stimulations in the network to which the individual belongs, rather than the indvidual's will.

Like molecules in a gas

Like neurons



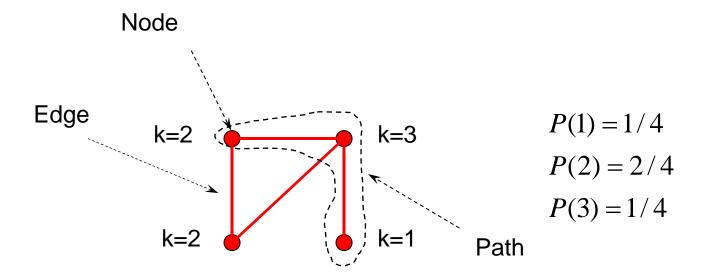
#### Camillo Golgi Nobel lecture, 1906

...far from being able to accept the idea of the individuality and independence of each nerve element, I have never had reason, up to now, to give up the concept which I have always stressed, that nerve cells, instead of working individually, act together [...]

However opposed it may seem to the popular tendency to individualize the elements, I cannot abandon the idea of a unitary action of the nervous system[...]

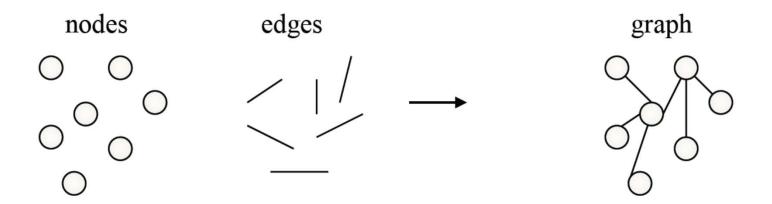
#### **Networks: Basic definitions**

1) A Network is a set of nodes connected by links (edges).



- 2) Degree (k) is the number of edges connected to a node
- 3) **Degree Distribution** P(k) is the fraction of nodes with degree k
- 4) Nodes can be linked directly by single edges or indirectly by sequences of intermediate nodes and edges: **paths**.

#### Graph theory

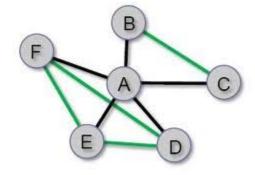


	train	internet	social	brain
node	station	server	person	brain regions
edge	rails	cables	relation	fiber/statistical dependency

#### The brain as a network

Interplay of (structural) segregation and (functional) integration.

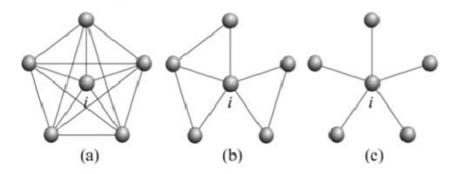
#### Local segregation: clustering



 $C_A = 4/10 = 0.4$ 

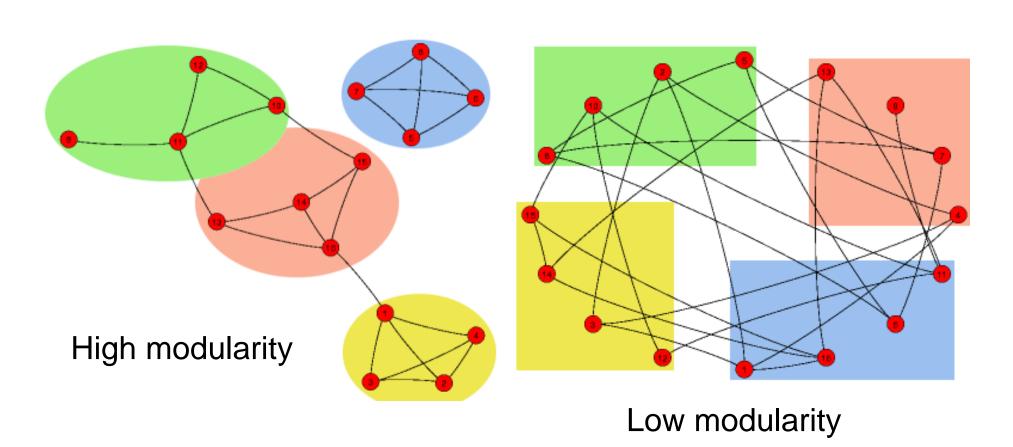
Fraction of existing links between neighbors over all possible links.

So, high clustering means connecting nodes that are well connected.



- a)  $cc_i = 10x2/5x4 = 1$ ;
- b)  $cc_i=3x2/5x4=0.3$ ;
- c)  $cc_i = 0x2/5x4 = 0$

# Local segregation: modularity (many links within modules, few links between modules)



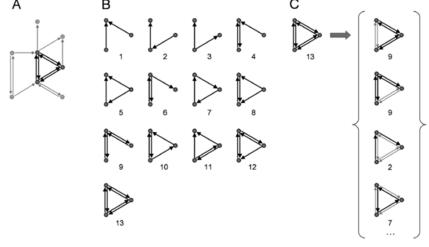
Newman & Girvan, PRE (2003)

#### Networks Motifs (Alon, 2003)

- Characteristic network building blocks
- Small connected subgraphs that occur significantly more frequently than in randomized networks
- Brain networks: small set of structural motifs, large number of functional motifs (Sporns, Bullmore)

#### **Identifying Network motifs**

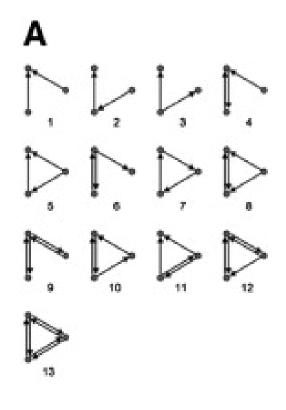
- Find n-node subgraphs in real graph.
- Find all n-node subgraphs in a set of randomized graphs with the same distribution of incoming and outgoing arrows. (Newman, 2000, Sneppen, Malsov 2002)
- Assign Z-score for each subgraph.
- Subgraphs with high Z-scores are denoted as Network Motifs.



13 possible subgraphs for 3 nodes

$$Z = \frac{N_{real} - \langle N_{rand} \rangle}{\sigma_{rand}}$$

# Structural motifs in the cortex across species



В

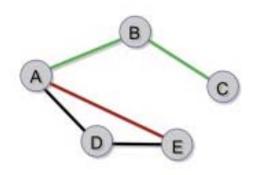
Brain Network	ID	Real	Random		
Human Cortex	13	N/A	N/A		
Macaque Visual Cortex	9	410	121.55 (21.03) z = 13.79		
Macaque Cortex	9	1833	223.66 (34.99) $z = 46.22$		
Cat Cortex	9	1217	472.33 (52.85) z = 14.16		
C. elegans	4	2999	1067.03 (121.52) z = 15.98		
	6	3415	1164.31 (134.71) z = 16.79		

### Structural and functional motifs in the cortex

Brain Network	М	Structural Motifs		Functional Motifs			
		Real	Random	Lattice	Real	Random	Lattice
Macaque Visual	2	190	243 (4)	191 (2)	432	380 (4)	431 (2)
Cortex	3	1,486	2,353 (51)	1,344 (40)	19,769	14,358 (325)	21,120 (308)
	4	10,487	18,076 (391)	8,688 (414)	1,843,308	1,013,131 (55,187)	2,259,970 (90,404)
	5	62,940	105,926 (2,059)	50,278 (2,863)	334,279,477	121,572,738 (13,874,054	513,004,042 (50,992,845
Macaque Cortex	2	438	654 (7)	471 (7)	1,054	838 (7)	1,021 (7)
	3	4,584	10,786 (227)	4,439 (143)	53,601	30,449 (648)	56,043 (871)
	4	51,129	173,235 (4,635)	39,345 (2,346)	5,306,188	1,850,355 (87,743)	6,617,493 (272,110)
Cat Cortex	2	519	656 (7)	510 (5)	1,054	838 (7)	1,021 (7)
	3	6,986	10,898 (160)	6,021 (122)	53,601	30,449 (648)	56,043 (871)
	4	87,673	149,791 (2,250)	65,527 (2,150)	5,306,188	1,850,355 (87,743)	6,617,493 (272,110)
C. elegans	2	1,718	1,922 (6)	1,700 (40)	2,230	2,026 (6)	2,248 (40)
	3	31,070	41,707 (279)	23,376 (1,494)	70,911	55,054 (363)	84,245 (4,200)
	4	674,125	1,081,682 (11,105)	316,228 (36,200)	3,430,885	2,160,611 (34,800)	5,326,201 (578,900)

Numbers are actual values (for real matrices) and mean and standard deviation (in parentheses, for random and lattice matrices, n = 100). DOI: 10.1371/journal.pbio.0020369.t001

## Global integration: path length and efficiency



Path length: number of connections that needs to be crossed to go from one node to another.

This measure is intuitively simple but varies greatly with size and density of graphs.

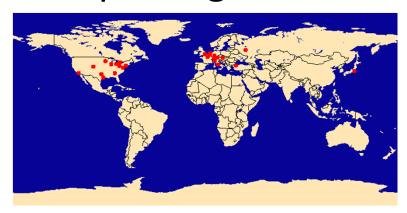
Efficiency: average of the inverse of the distances.

Segregation and integration place opposite demands on networks:

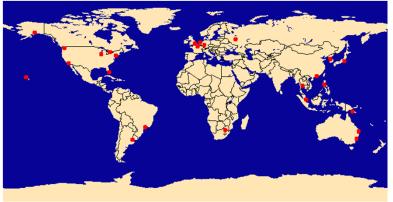
- Optimal clustering and modularity are inconsistent with high integration (little cross-talk among highly segregated communities)
- Optimal efficiency or integration is only achieved in a fully connected network that lacks any differentiation in its local processing
- The bridge between these two opposite requirements is made by heterogeneous contributions by individual nodes and edges.

#### Influence and centrality: hubs

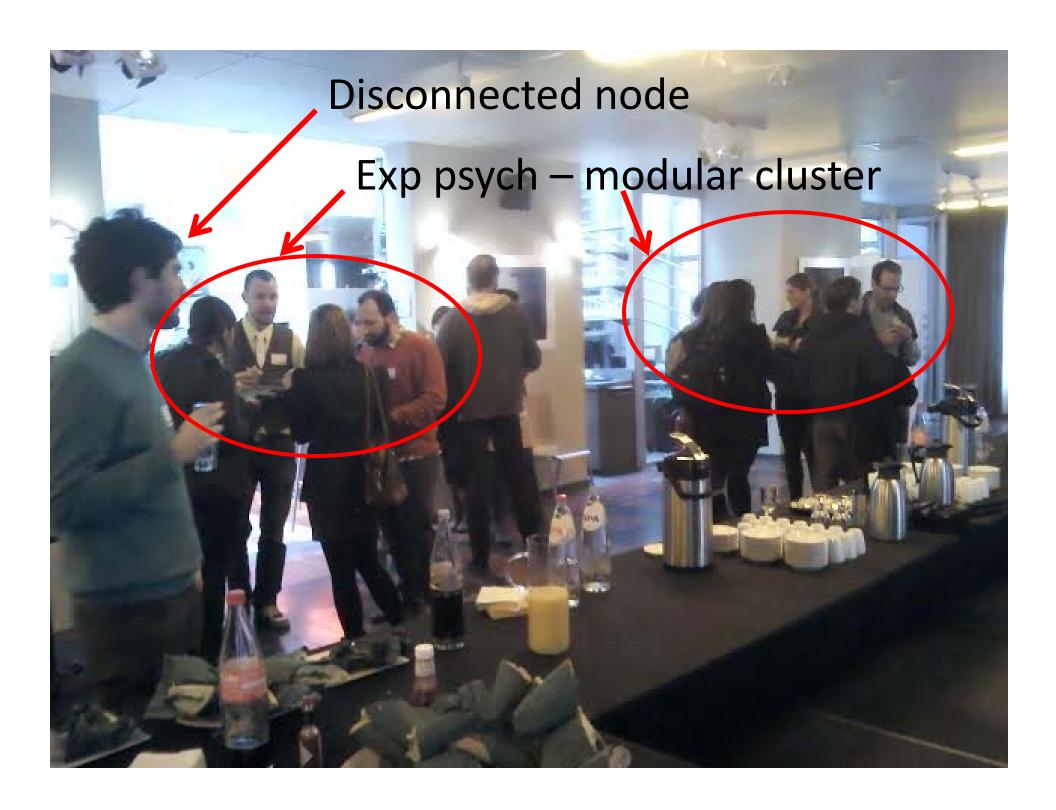
- The number of connections is not enough to quantify the importance of a node
- Centrality: fraction of short path length passing from a node.



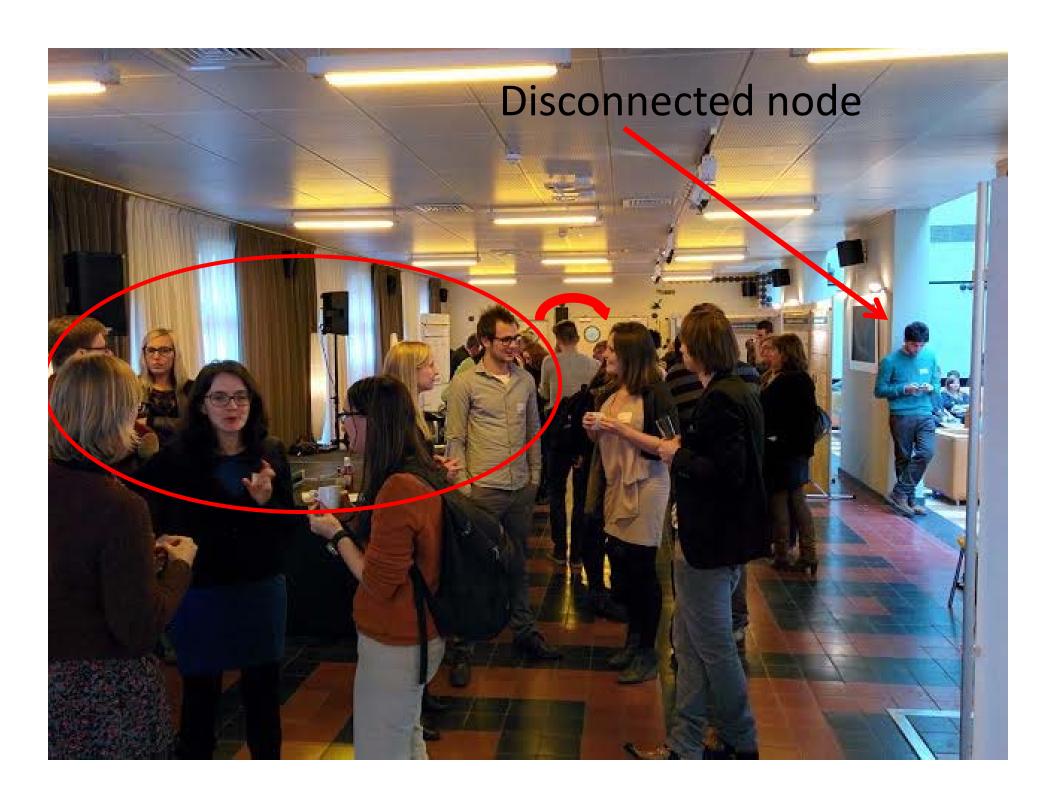
Most connected airports

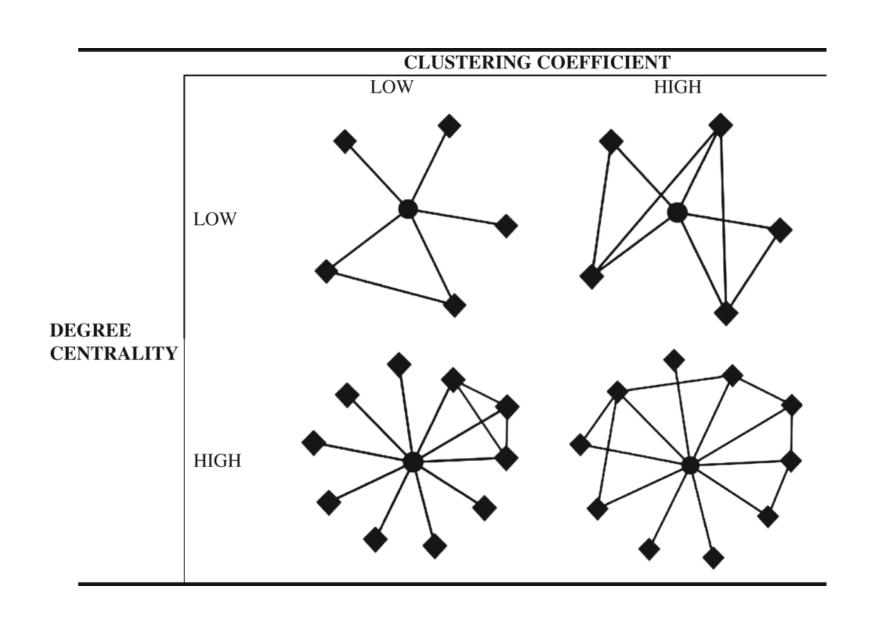


Most central airports

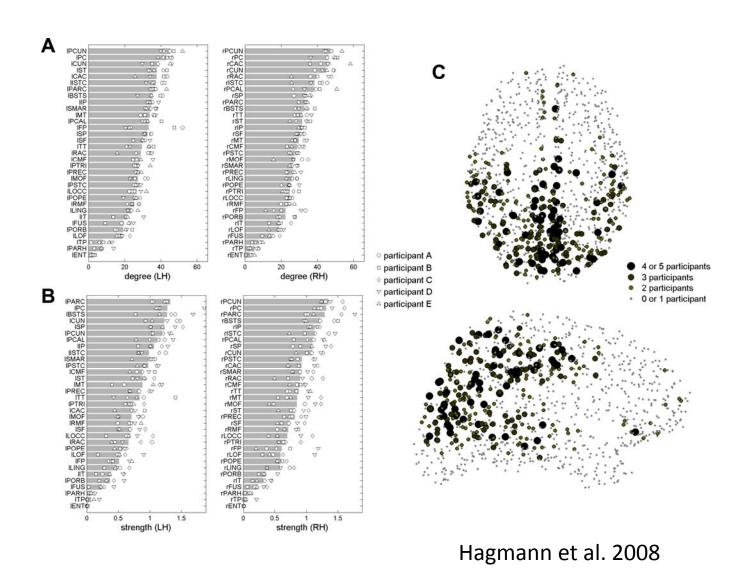




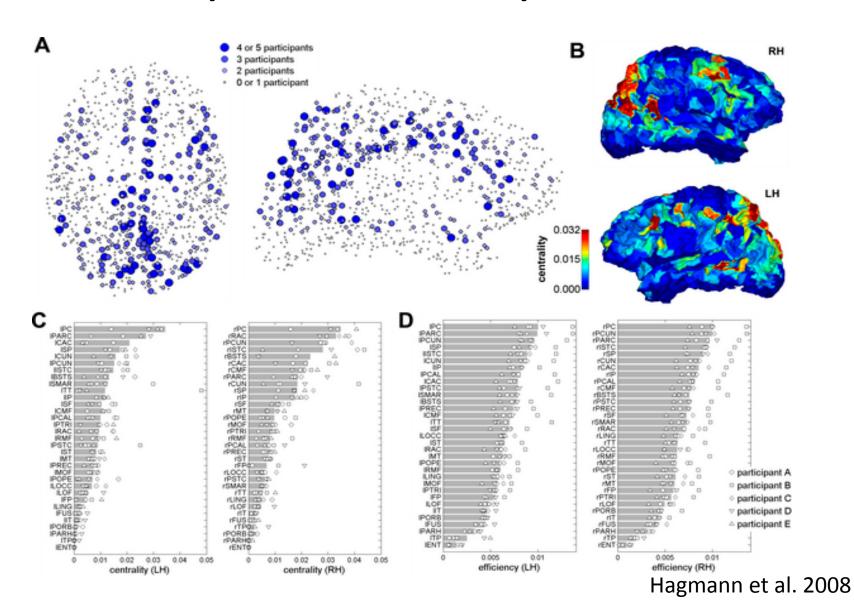




#### Node degree and strength in the brain



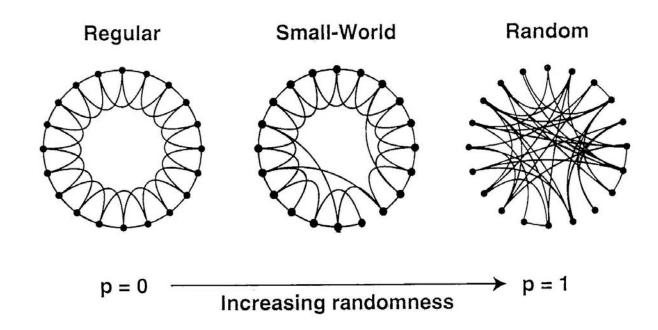
#### Centrality and efficiency in the brain

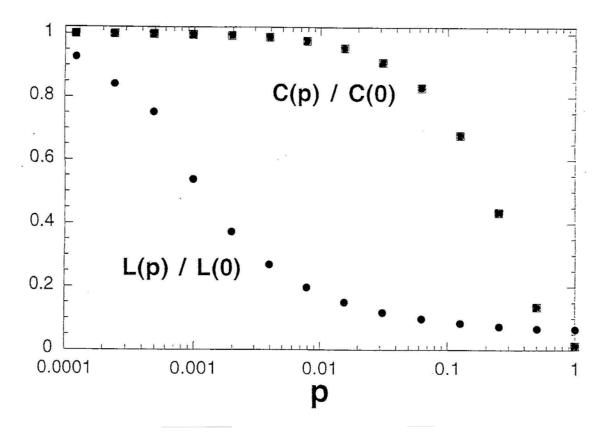


## Network architectures: order, disorder, hierarchy

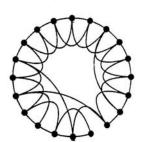
The architectural features of a graph reflect the processes by which the graph was constructed or developed.

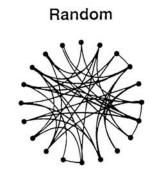
### From regular to random





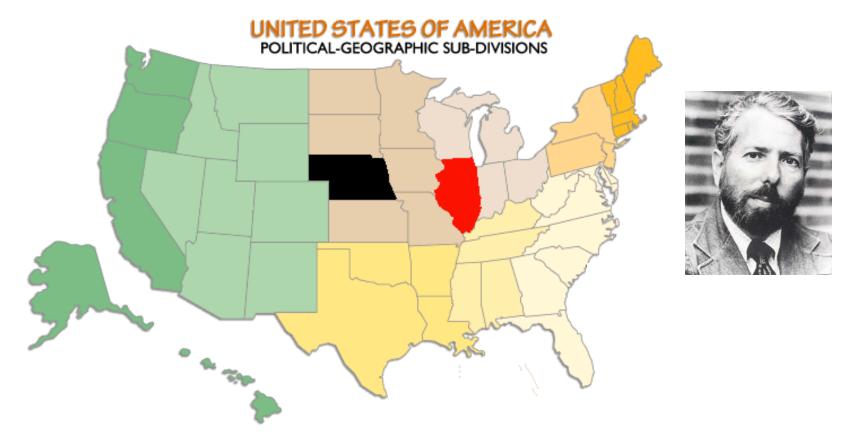
Regular





Small-World

#### Small world effect: Milgram (1967)



Will a message arrive from a random Nebraska location to a clerk in Chicago, through hand-to hand passage?

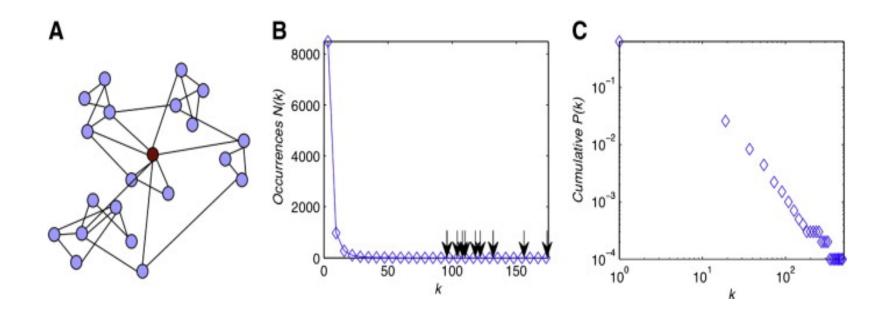
As you might have understood, a lot of networks are small world

Sometimes small-worldness can arise from nonprofound mechanisms: i.e. the simple effect of using a (thresholded) correlation to obtain the links of a network, makes this network a small-world one (Zalesky et al. 2012, Hlinka et al. 2012)

# What makes inter-community crossing possible?

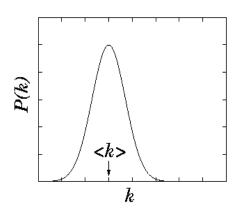
1.Weak, random ties2.Hubs

### Signature of hubs in degree distribution



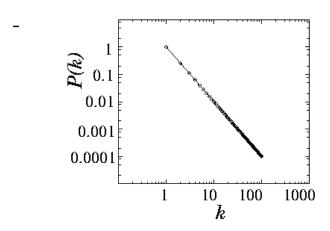
#### Hierarchical vs symmetric networks

#### Homogeneous network

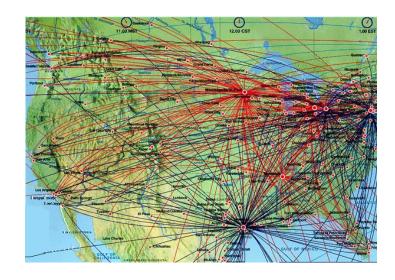




# Hierarchical network: hubs

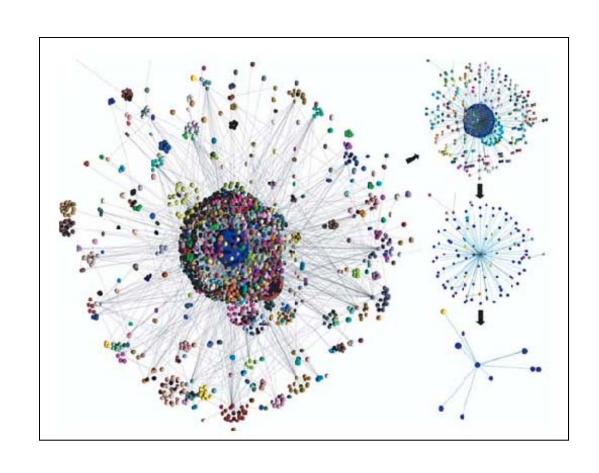


#### **Scale-free**



#### Self-similarity of scale free networks

 $P(k) \sim k^{-\gamma}$ 



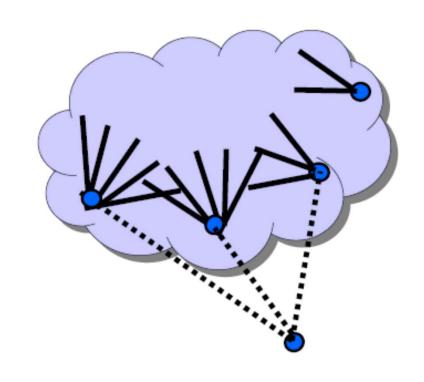
WWW (in)	Internet	Actor	Citation index	Sex Web	Cellular network	Phone call network	linguistics
$\gamma = 2.1$	$\gamma = 2.5$	$\gamma = 2.3$	$\gamma = 3$	$\gamma = 3.5$	$\gamma = 2.1$	$\gamma = 2.1$	$\gamma = 2.8$

# What is the mechanism originating scale-free networks? PREFERENTIAL ATTACHMENT (Albert & Barabasi, 1999)

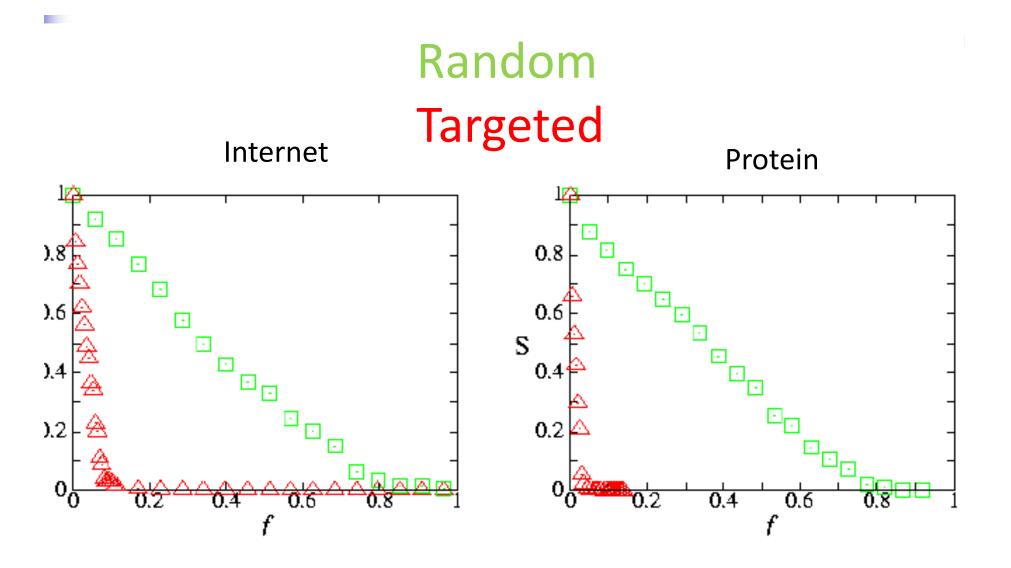
#### Matthew effect

For unto every one that hath shall be given, and he shall have abundance: but from him that hath not shall be taken even that which he hath.

Matthew 25:29



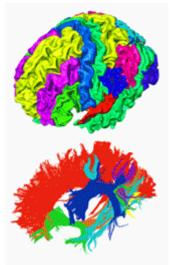
#### Robustness wrt casual or targeted attacks

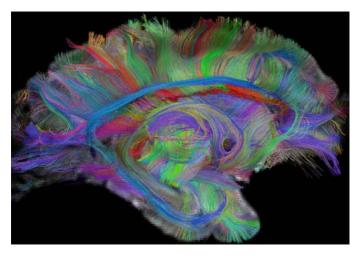


#### Networks in the brain

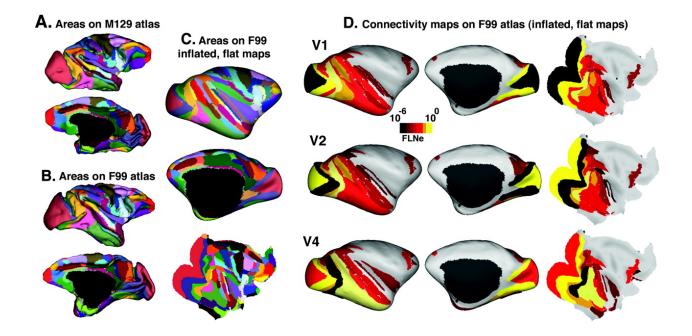
- Which connections exist in the brain?
- Is it possible that regions that are not connected by neural fibers still communicate?
- Is it possible to detect the flow of information in the brain?
- Which properties has this network of communications?
- How does this network change during task performing vs rest, or disease vs health?

# Anatomic or structural connectivity (at large scale)





DTI (humans)



Dye tracing (animals)

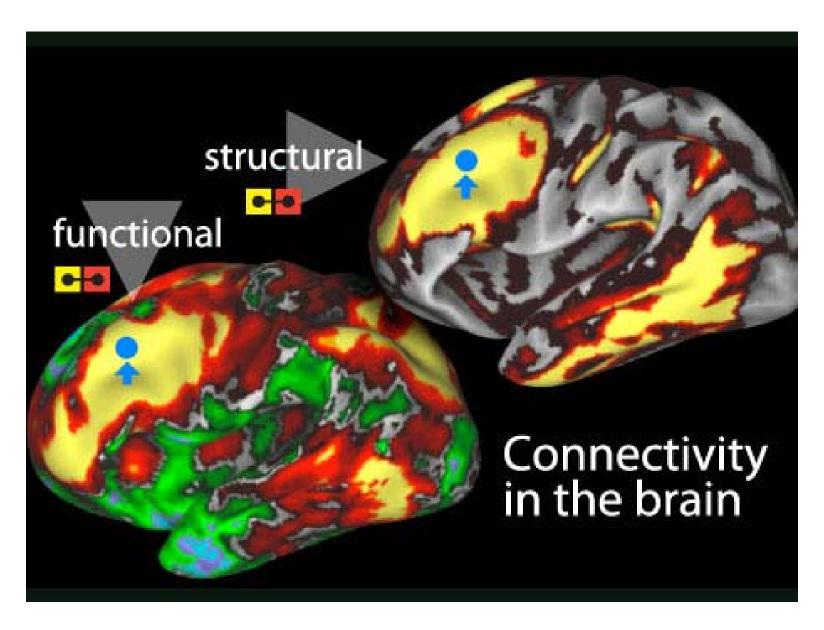
#### Functional (dynamical) connectivity

Find statistical dependencies between even remote regions:

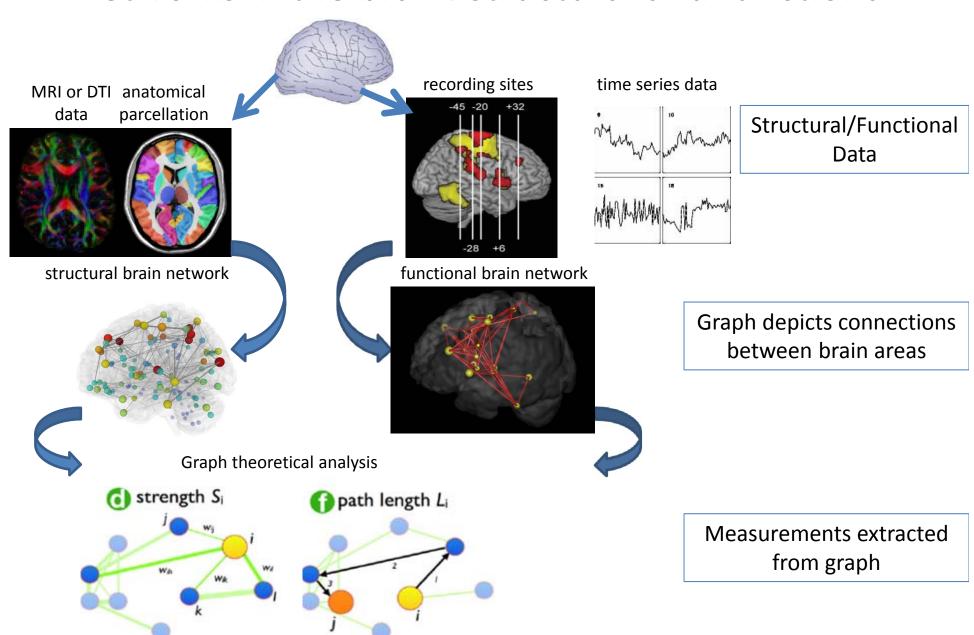
- Correlation
- Coherence
- Phase synchronization
- Mutual information

• ...

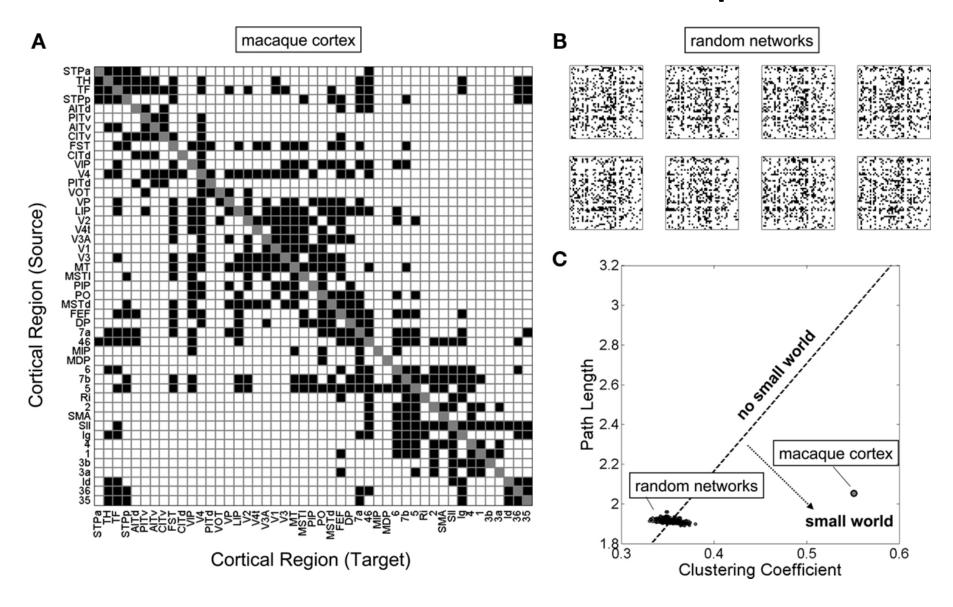
#### Functional vs structural



#### Networks in the brain: structural and functional



### Where are cortical maps?



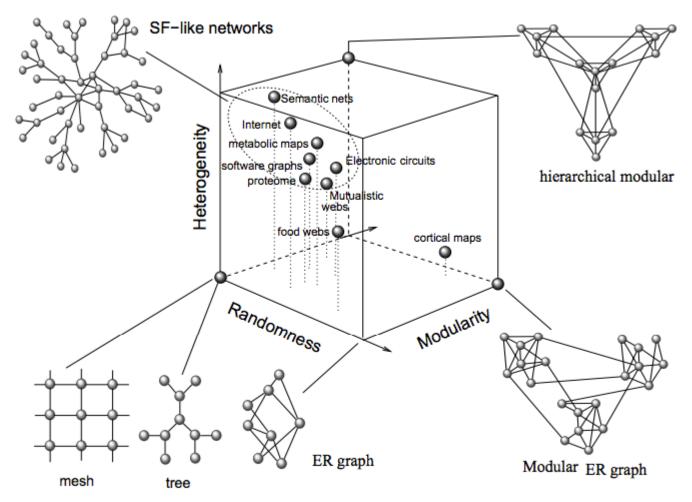
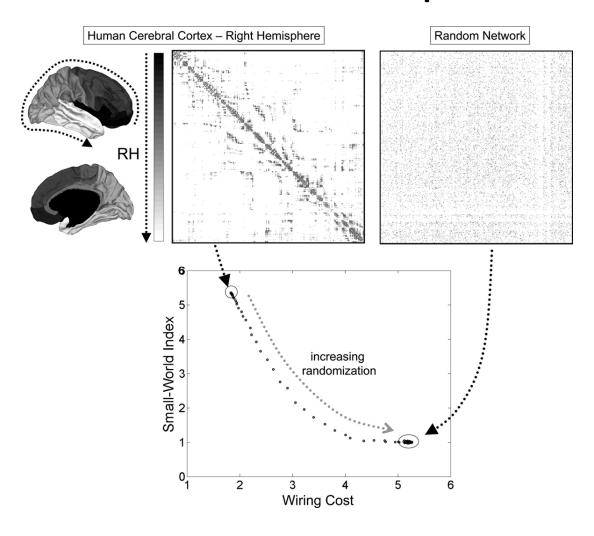


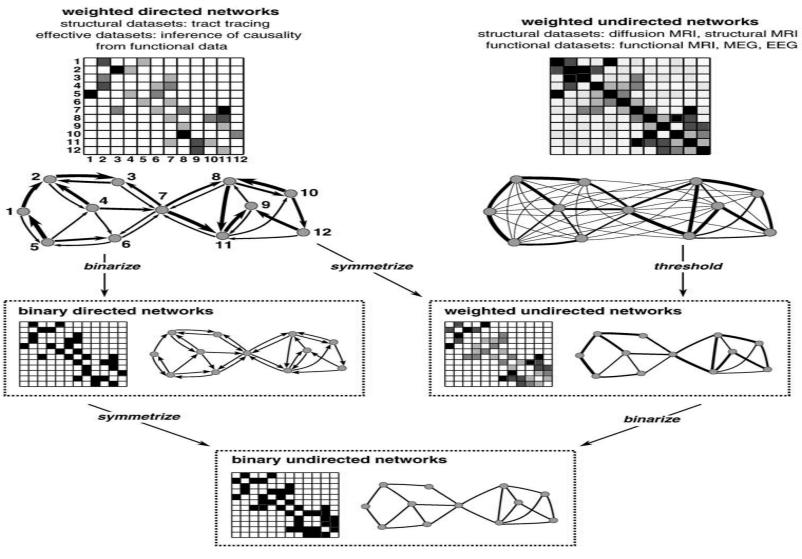
FIG. 3 A zoo of complex networks. In this qualitative space, three relevant characteristics are included: randomness, heterogeneity and modularity. The first introduces the amount of randomness involved in the process of network's building. The second measures how diverse is the link distribution and the third would measure how modular is the architecture. The position of different examples are only a visual guide. The domain of highly heterogeneous, random hierarchical networks appears much more occupied than others. Scale-free like networks belong to this domain.

# Small-world topology allows for lower resource consumption

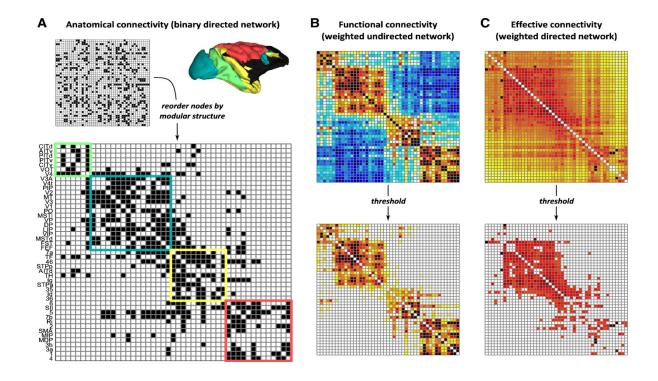


Network modality	Edge representation	Empirical techniques	Network characteristics
Structural connectivity	Physical links (synapses, pathways), biophysical efficacy, time delay	Microscopy: tissue volume reconstruction Neuroanatomy: tract tracing Neuroimaging: diffusion imaging/tractography	Weighted or unweighted, sparse and directed (synapses, projections), sparse and undirected (diffusion MRI)
Functional connectivity	Statistical relationships between neural time courses (e.g. spikes, EEG, BOLD)	Neurophysiology: spike or LFP correlations EEG/MEG: correlation, sync, coherence, phase locking fMRI: BOLD cross-correlations, partial correlations	Full and weighted, or sparse and weighted (or unweighted) after thresholding; undirected
Effective connectivity	Causality inference based on temporal precedence or on generative model	Spikes, EEG/MEG, fMRI: time series analysis (Granger causality, Transfer entropy) or model inference (dynamic causal modeling)	Full or sparse; weighted or unweighted; directed

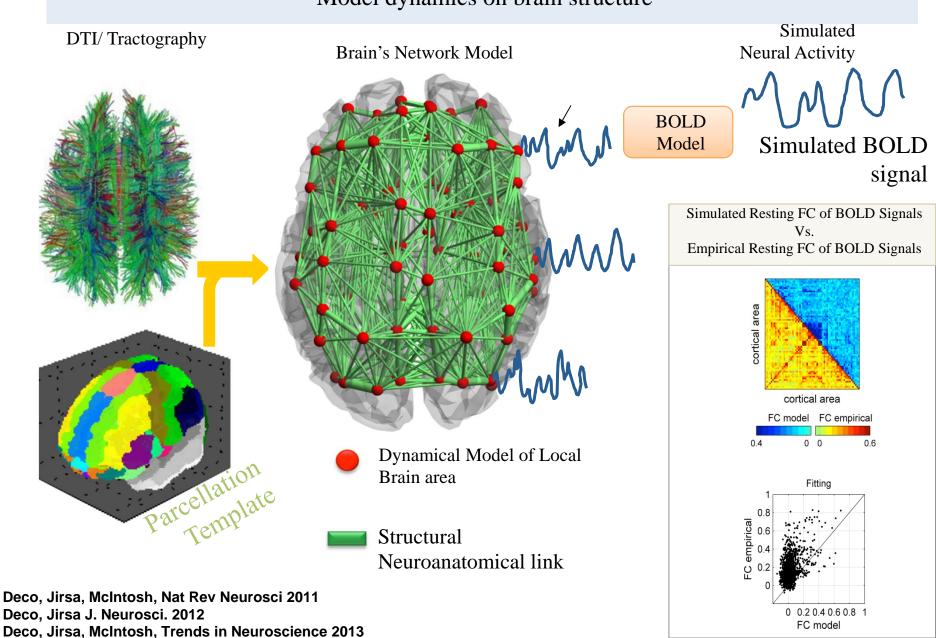
#### From data to network



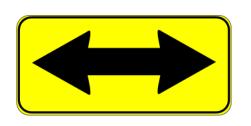
#### From data to network



#### Model dynamics on brain structure



# Network inference from temporally correlated data



Correlations Coherence Phase synchronization Generalized synchronization Mutual information Transfer entropy **Granger causality** 

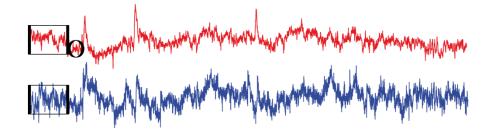


#### Predicting the future of a time series

Using only its past...

$$x = AX + \varepsilon_X$$

... or including the past of another time series



$$x = B[X Y] + \varepsilon_{X,Y}$$

$$\varepsilon_{X,Y} < \varepsilon_X \rightarrow Y$$
 Granger-causes X

#### Granger causality and Transfer Entropy

GC and TE are equivalent for Gaussianly distributed variables and other quasi-Gaussian distribution (Barnett et al. 2009, Hlavackova 2011, Barnett and Bossomaier 2012)

In this case they both measure information transfer

- Unified approach
- Mathematically more treatable
- You make heavy assumptions and you could lose important features

Establishment of a general framework for GC and TE, which computations that can be both exact and approximate

### Advances for (fMRI) connectivity

Many variables, few samples

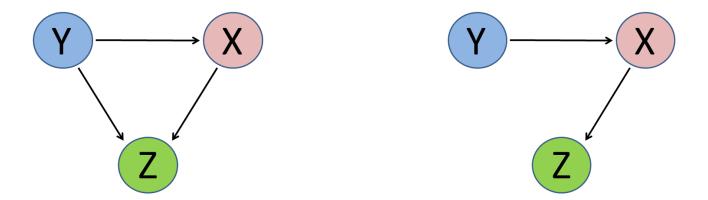
Confounding HRF effect



Bad temporal resolution

#### GC in multivariate datasets

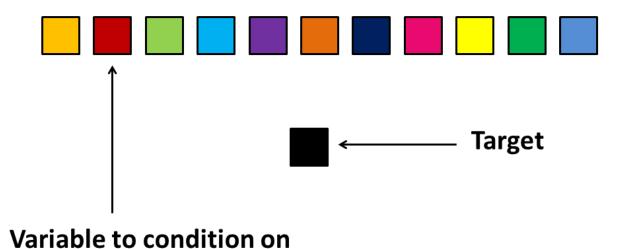
We must condition GC to the presence of other variables



This problem has been known from the start, and the solution is usually the conditioned approach (Geweke 1982)

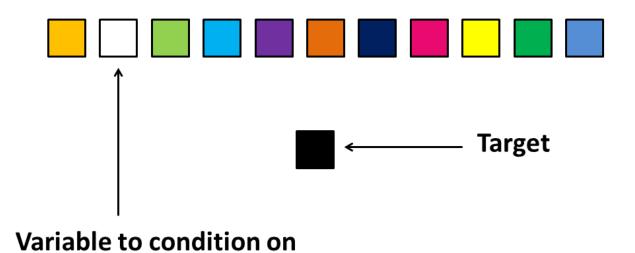
#### Full conditioning in multivariate dataset

We compare the model including ...



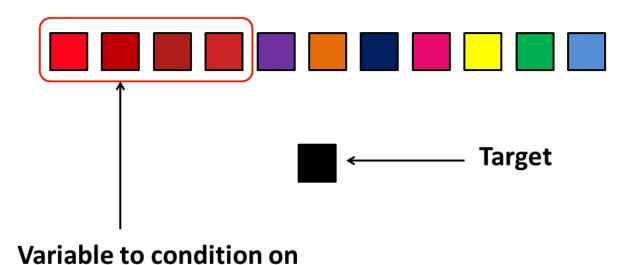
#### Full conditioning in multivariate dataset

... and excluding the conditioning variable



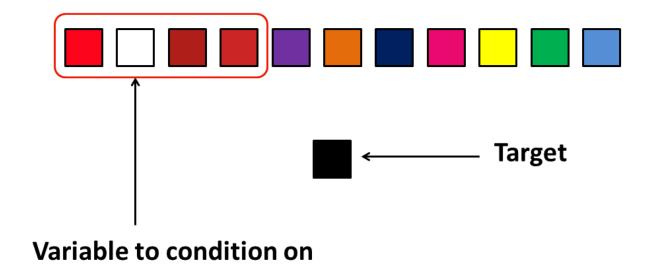
#### Presence of redundancy

When a number of variables share the same info on the target ...



#### Presence of redundancy

... in the model we still have info on the target

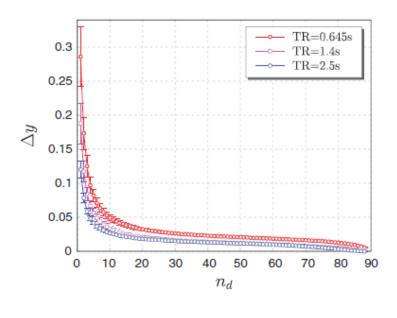


and the conditioning variable and all those correlated with it will be regarded as not relevant

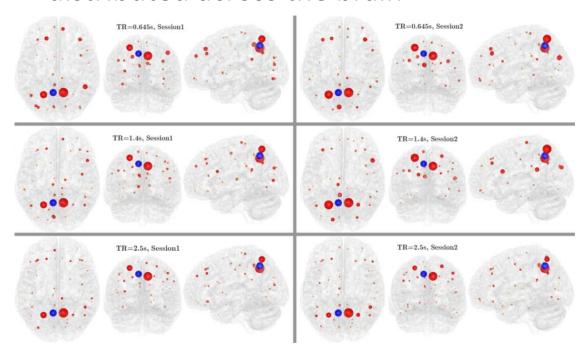
#### Partially conditioned Granger causality

- Redundancy in multivariate datasets leads to false GC estimations
- Conditioning on the most informative variables for each candidate driver

#### Residual information gain



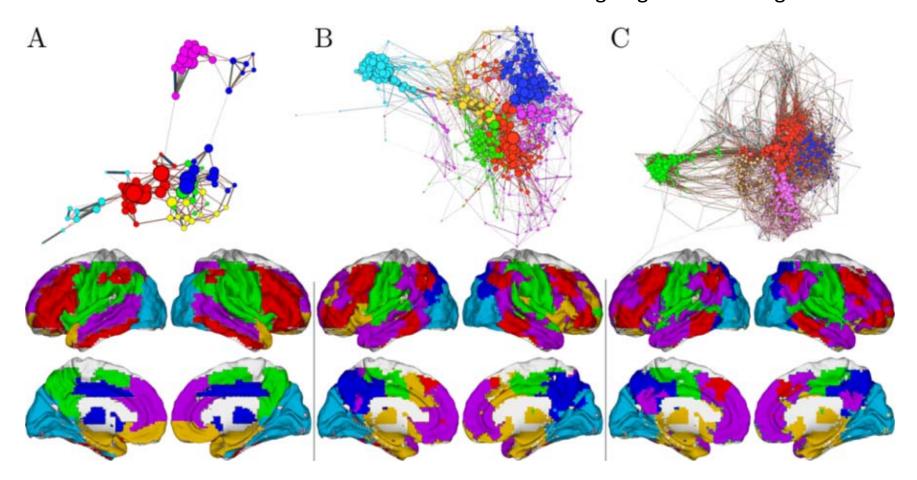
## Most informative regions consistently distributed across the brain



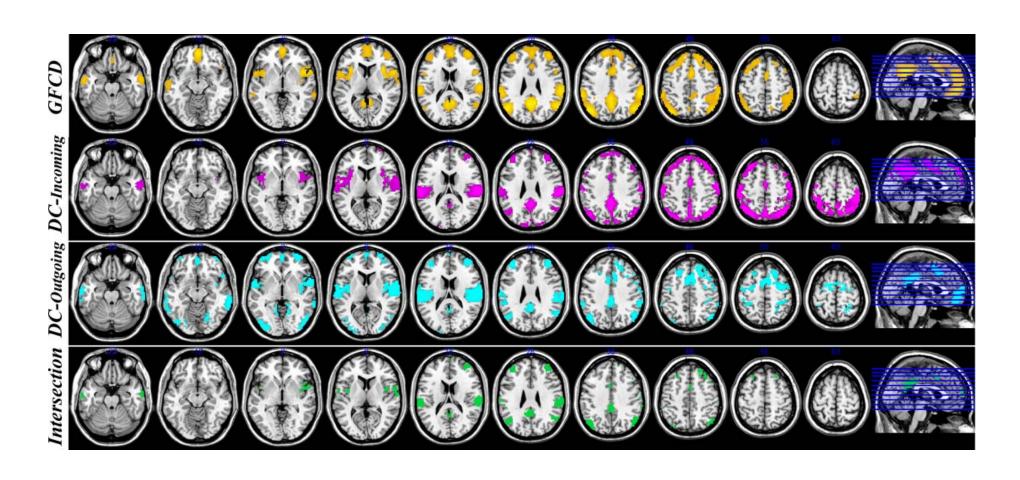
Marinazzo et al, Computational and mathematical methods in medicine, 2012 Wu et al. Brain Connectivity 2013

#### From regional to voxel level

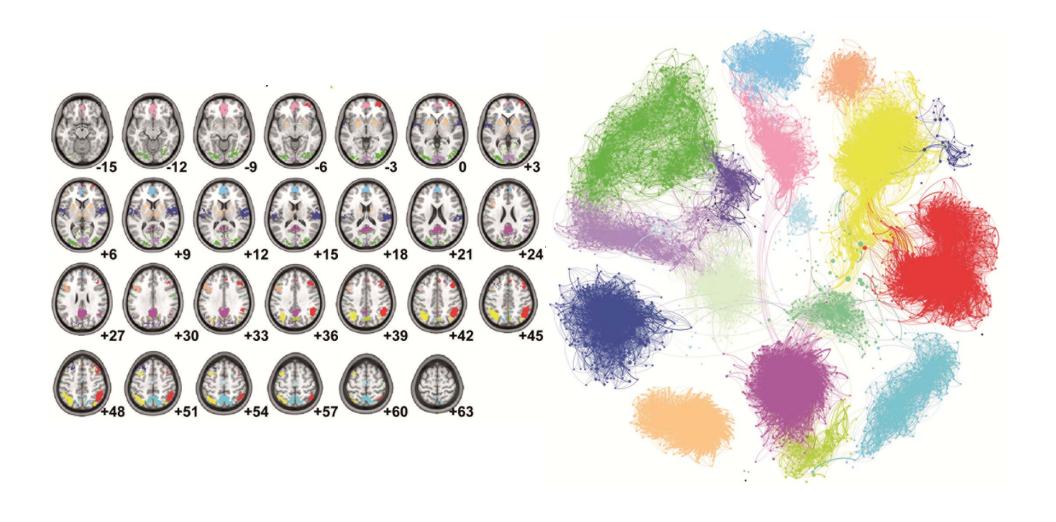
The modular structure of brain networks can as a prior for further dimensionality reduction Reconstruction of voxel-wise directed networks: hubs for outgoing and incoming information



### Connectivity density

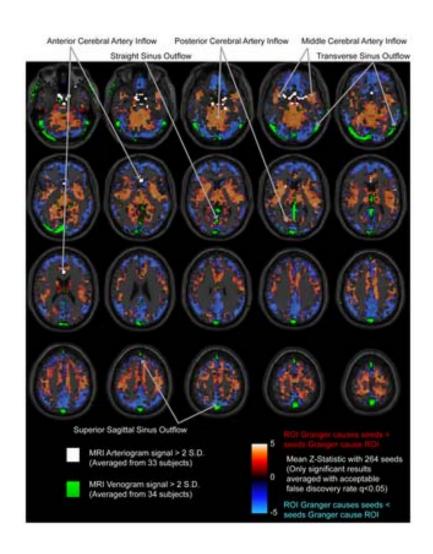


#### Connectome at voxel level



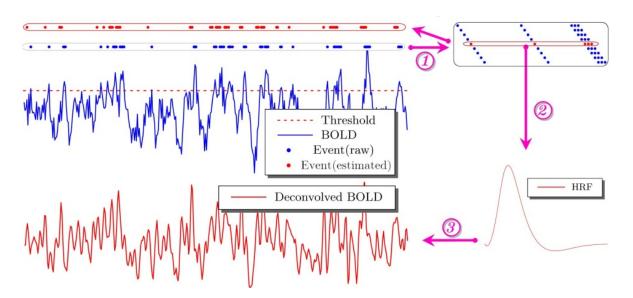
#### The importance of deconvolution

Alard Roebroeck's talk today, discussion on David et al. 2008



Anderson et al. PLOS One, 2013

#### Point processes in BOLD signal

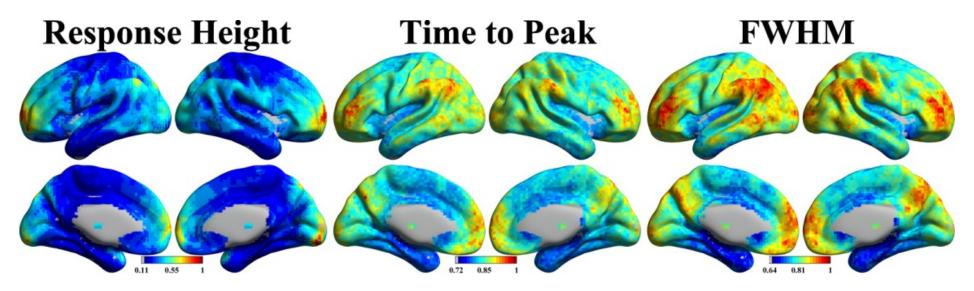


Peak events in BOLD time series can be considered as neural pseudoevents.

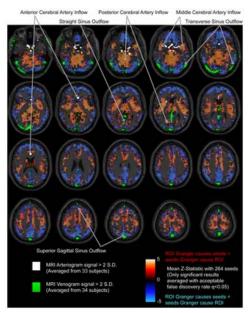
Delay from event to BOLD peak by error minimization

HRF reconstructed as canonical, FIR, or rbeta

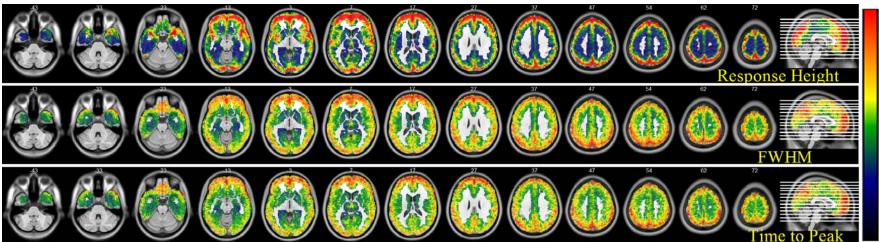
Map HRF parameters across the brain

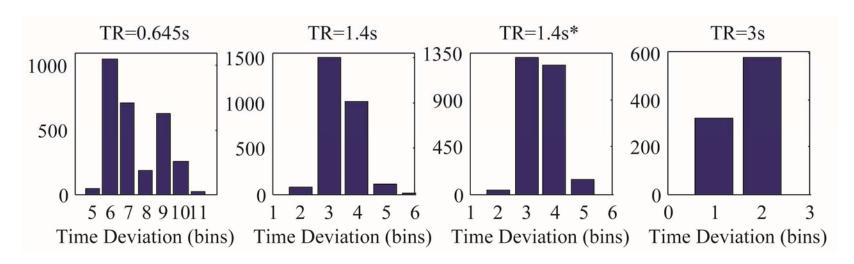


### The importance of deconvolution

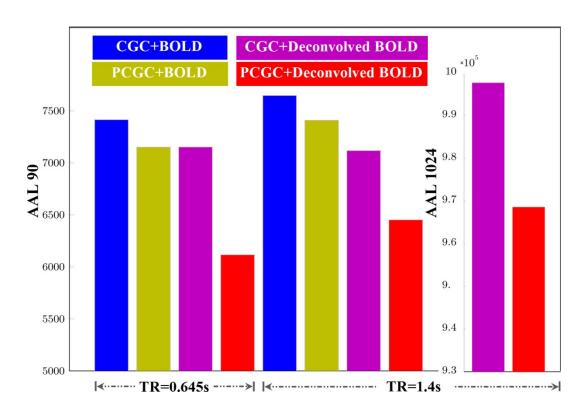


http://figshare.com/articles/HRF\_parameter/886139





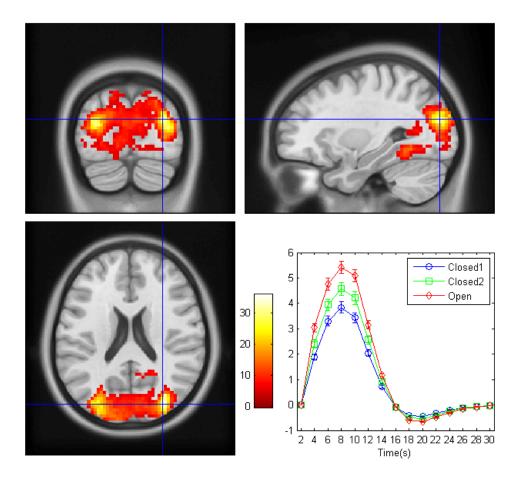
Distribution of delays between neural onset and bold peak



Decreased variance of results with PCGC + deconvolution

#### HRF shape as a marker of brain function

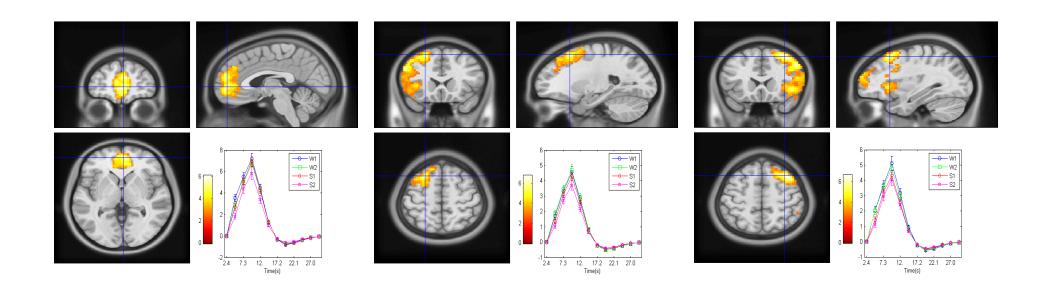
Eyes closed, then open, then closed again



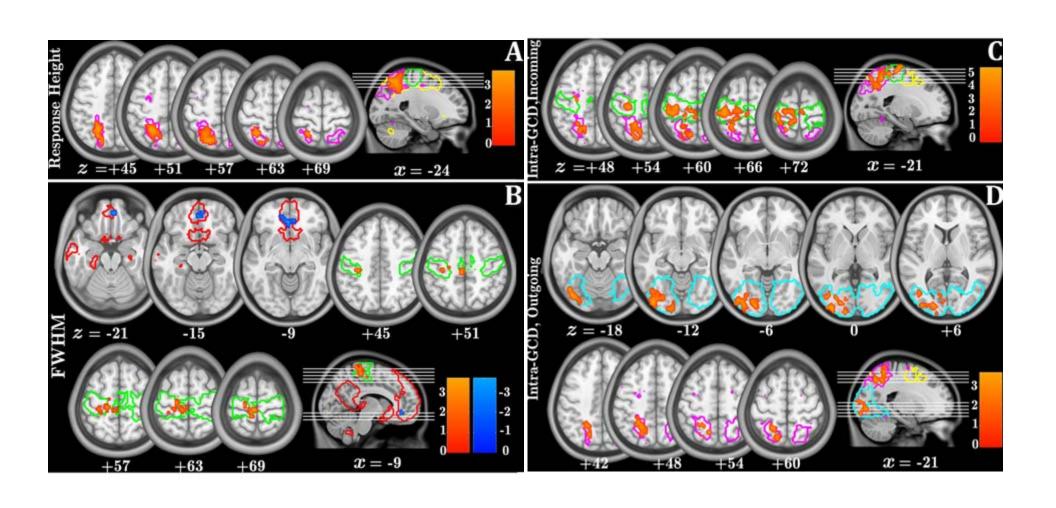
#### HRF shape as a marker of brain function

Propofol anesthesia

Wake -> Mild sedation -> Deep sedation -> Recovery of consciousness

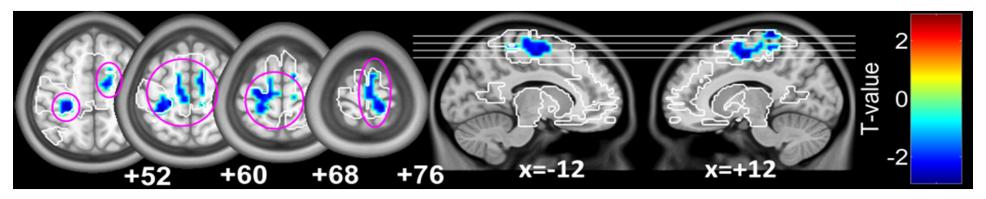


# HRF shape and GCD in left handers vs right handers

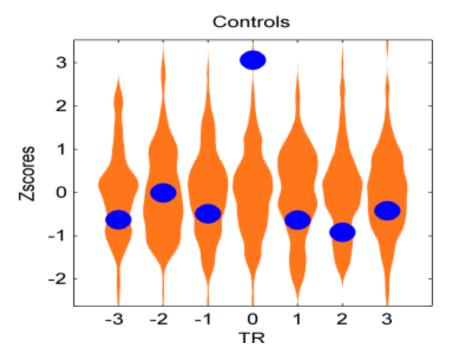


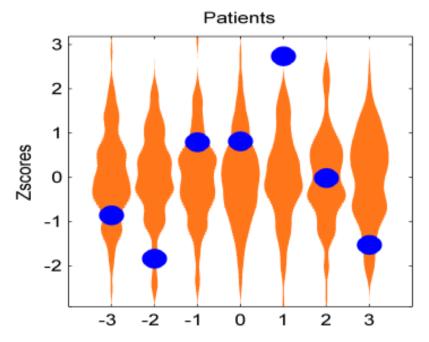
### Connectivity from point processes

Probability of an event in the target region after an event in the driver region



Nucleus accumbens-pain matrix: correlation in controls, lagged influence in chronic back pain





#### Tools

- GCCNT Toolbox (Wu)
- BCT Toolbox (Rubinov and Sporns)
- GAT Toolbox (Hosseini)
- Brainnetviewer (Xia)

#### The C-word curse

"Every decade or so, a grandiose theory comes along, bearing similar aspirations and often brandishing an ominous-sounding C-name. In the 1960 it was cybernetics. In the '70s it was catastrophe theory. Then came chaos theory in the '80s and complexity theory in the '90s."

Steven Strogatz, Sync

- Correlation
- Causality
- Connectivity