Model-Based fMRI Analysis

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- Motivation
  - Models (general)
  - Why you ought to care
- Model-based fMRI
  - Models (specific)
  - From model to analysis
- Extended Example
  - Hampton, Bossaerts, & O'Doherty (2006)

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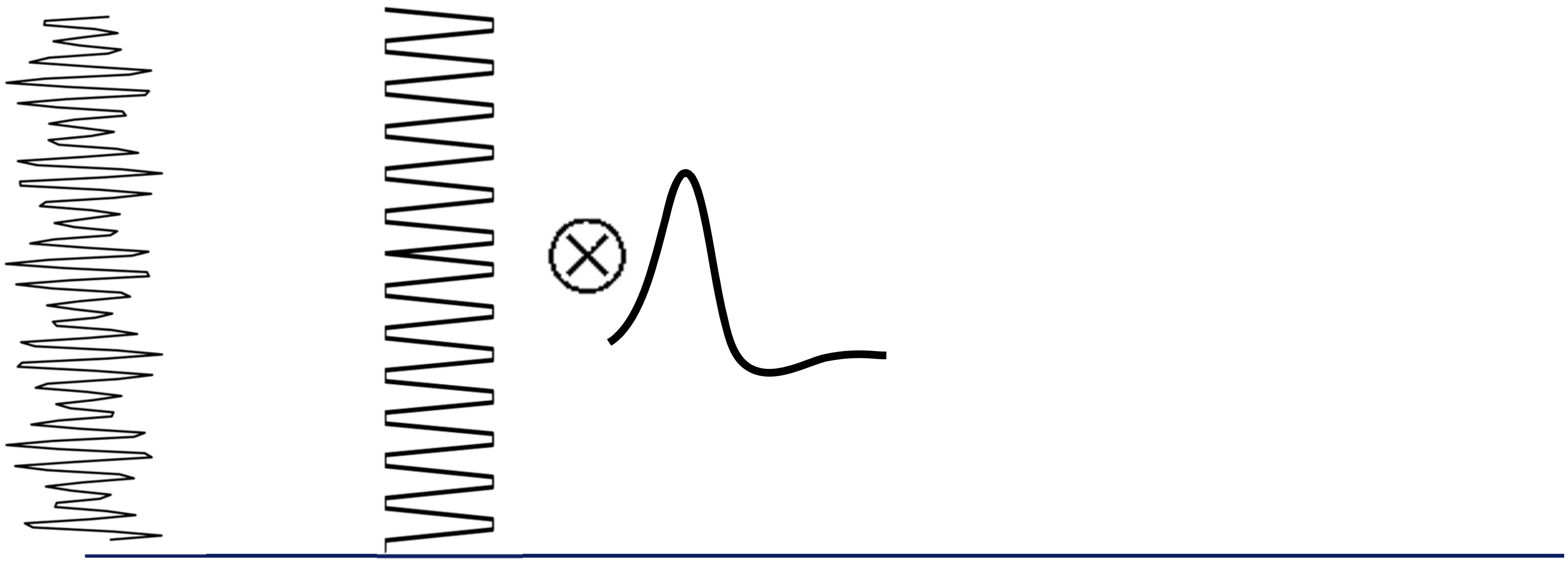
You probably already use a model

Trivial sense

General Linear Model

Y	**=**	**X**	**.**	**β**	**+**	**ε**
BOLD signal		Design Matrix convolved with HRF		Contribution of X to Y		Error



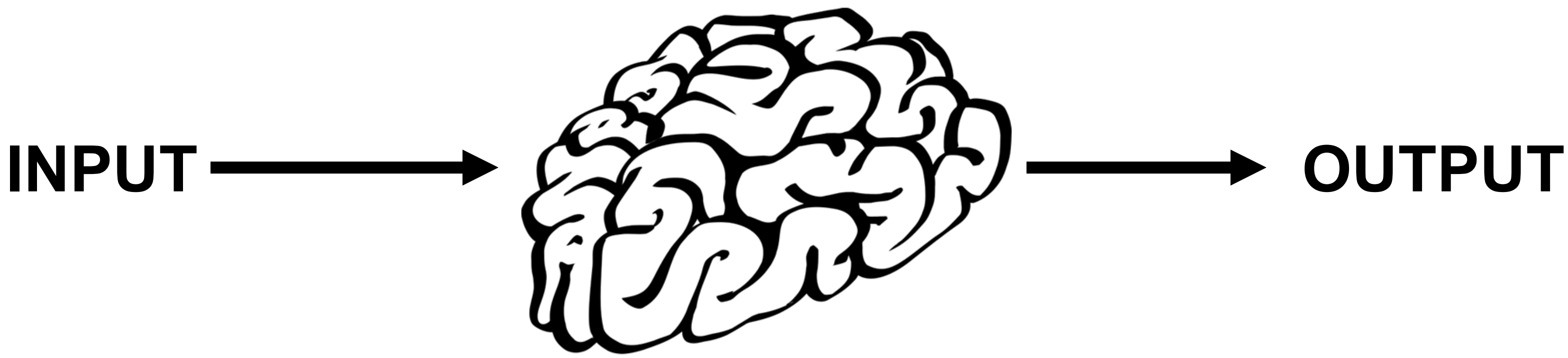
Design matrix is a model of an experiment

- May not be the best model
- Model-free analyses
  - ICA, clustering, etc

Model of BOLD signal

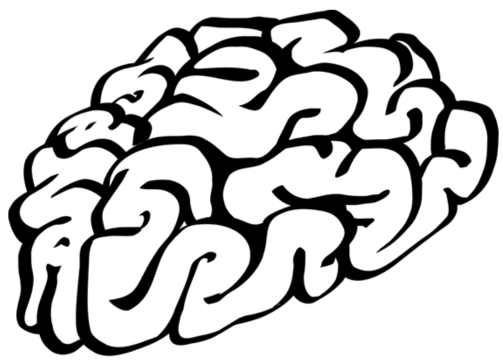
-HRF may vary from trial to trial, or from region to region

Less trivial sense



INPUT

visual
auditory
affective
cognitive

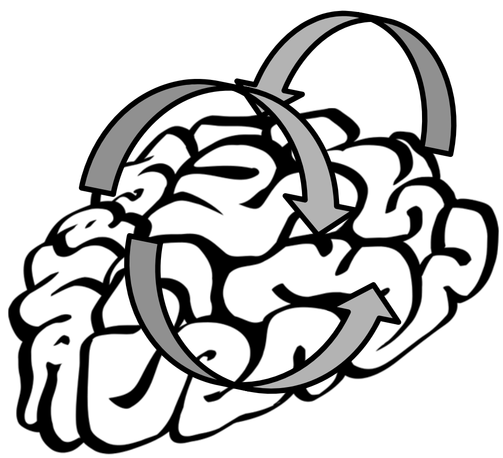


occipital
temporal
insula
prefrontal

OUTPUT

behavioral
neural activity
autonomic

INPUT
visual
auditory
affective
cognitive

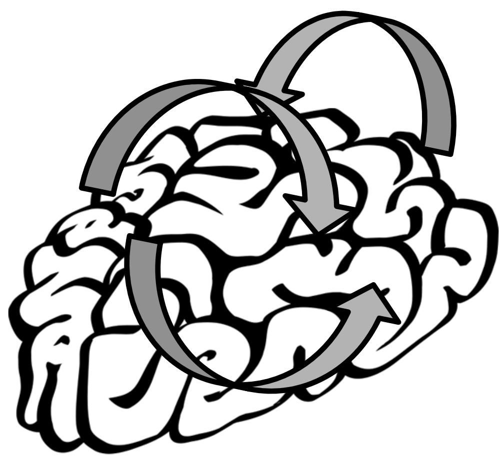


occipital
temporal
insula
prefrontal

OUTPUT
behavioral
neural activity
autonomic
responses

What do the arrows mean?

INPUT
- visual
- auditory
- affective
- cognitive



- occipital
- temporal
- insula
- prefrontal

OUTPUT
- behavioral
- neural activity
- autonomic responses

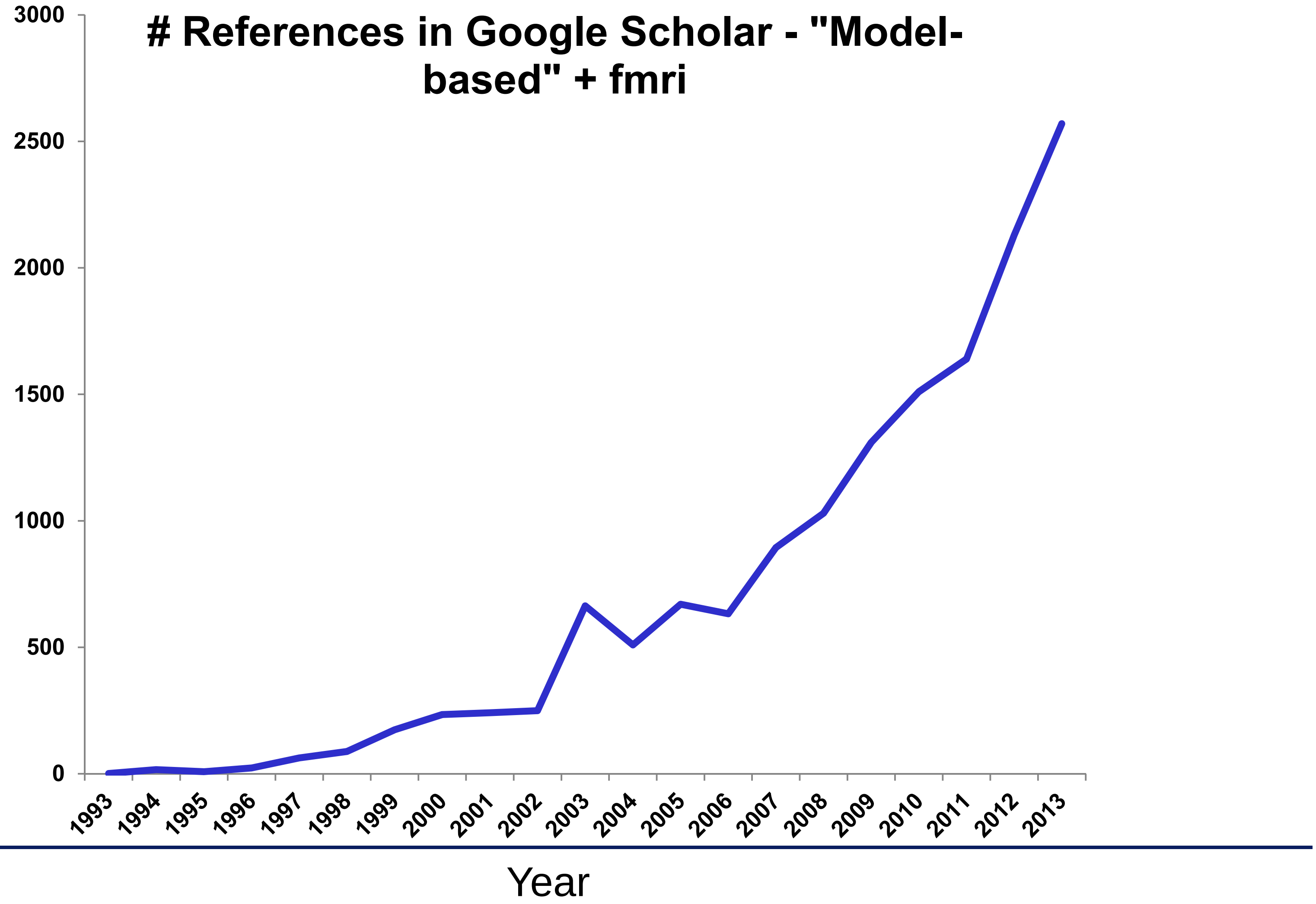
Model-Based fMRI analysis

Computational/mathematical models

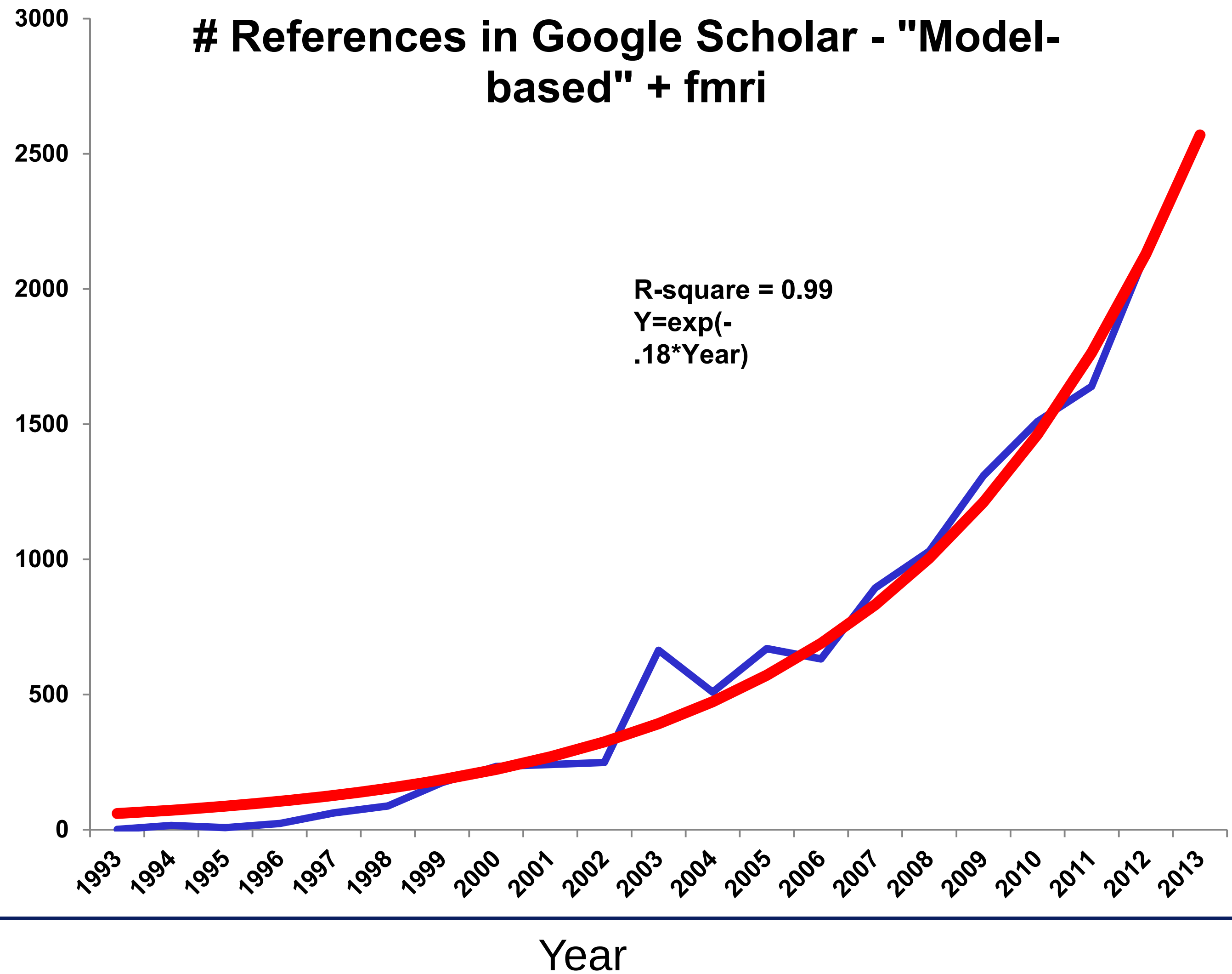
Target behavior
Mathematical formalization
Specify underlying mechanism

Why should we care?

Appeal to Self Interest



Appeal to Self Interest



Appeal to Abstract Principles

Scientific progress depends on making and testing predictions

Quantifiable predictions are more amenable to testing

No perfect models, but there are less imperfect models

Appeal to Abstract Principles II

Explanation vs. Description

Specification of mechanism

Understanding rather knowing

Appeal to Self Interest II

A good model allows you to predict the future.

Saves work

Identify interesting research questions (before everyone else)

- Motivation
  - Models (general)
  - Why you ought to care
- Model-based fMRI
  - Models (specific)
  - From model to analysis
- Extended Example
  - Hampton, Bossaerts, & O'Doherty (2006)

Goal of Model-Based fMRI analysis

Identify where/whether/how a particular cognitive mechanism, as specified by a computational/ mathematical model, is implemented in the brain.

Steps in Model-Based Analysis

1. Select/create a computational model
2. Design experimental task in order to test model
3. Fit model to behavioral data
4. Generate model predictions of brain activity
5. Regress model predictions against fMRI data

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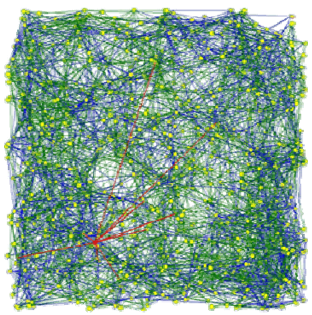
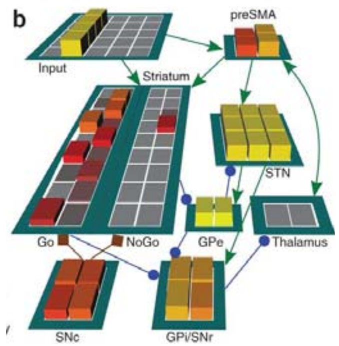
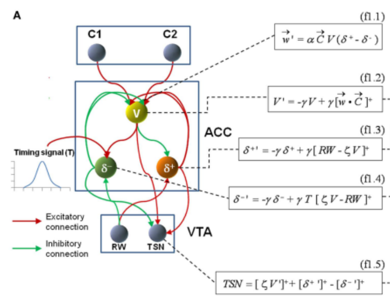
Types of models

Spiking Neural Networks	Dynamical Systems Models	RL/Connectionist Models	Mathematical/ Bayesian Models







$$P(s_k = 1|\mathrm{s}_{k-1})$$
$$= \int P(s_k = 1|r_k)p(r_k|\mathrm{s}_{k-1})dr_k = \int r_k p(r_k|\mathrm{s}_{k-1})dr_k = \langle r_k|\mathrm{s}_{k-1}\rangle.$$

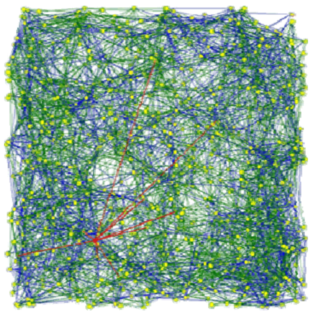
$$p(r_k|\mathrm{s}_{k-1}) = \alpha p(r_{k-1}|\mathrm{s}_{k-1}) + (1-\alpha)p_0(r_k),$$
$$p(r_k|\mathrm{s}_k) \propto P(s_k|r_k)p(r_k|\mathrm{s}_{k-1}).$$

Concrete → Abstract

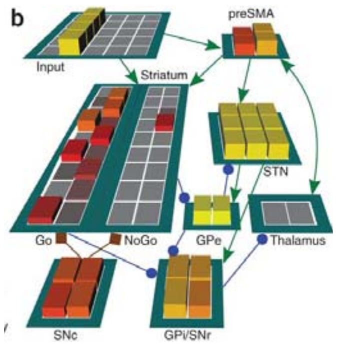
Spiking Neural Networks	Dynamical Systems Models	RL/Connectionist Models	Mathematical/ Bayesian Models
Neurobiologically Realistic	Neurobiologically Plausible	Related to Brain Structure	Passing Acquaintance with the Brain
Activity of Individual Neurons	Activity of Groups of Neurons	Activity of Brain Regions	Networks of Brain Regions

Types of models

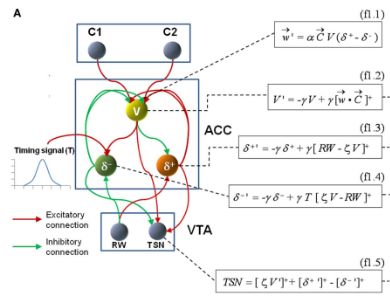
Spiking Neural Networks



Dynamical Systems Models



RL/Connectionist Models



Mathematical/ Bayesian Models

$$P(s_k = 1|\mathbf{s}_{k-1}) = \int P(s_k = 1|r_k)p(r_k|\mathbf{s}_{k-1})dr_k = \int r_k p(r_k|\mathbf{s}_{k-1})dr_k = \langle r_k|\mathbf{s}_{k-1}\rangle.$$

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Concrete → Abstract

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Example Models

Reinforcement Learning

e.g., Rescorla & Wagner (1972)

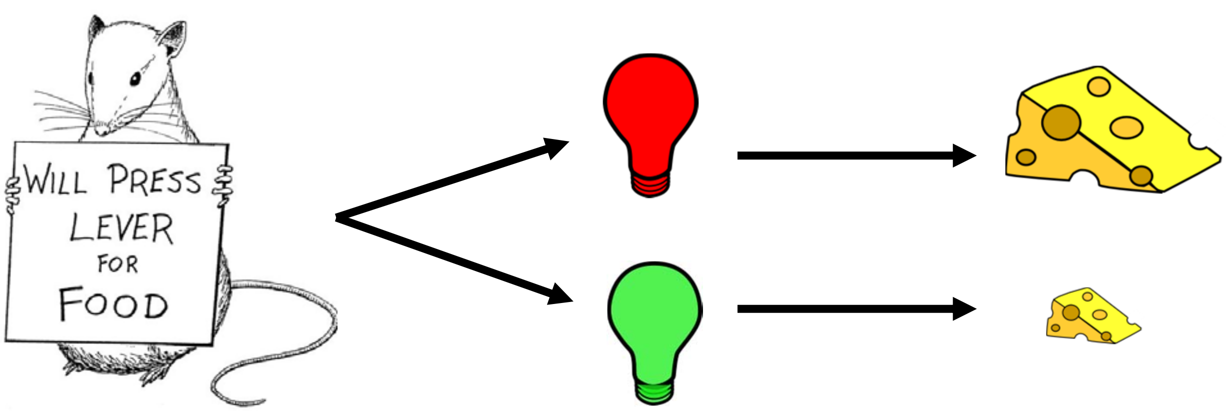
-Behavioral output

Choice

-Cognitive Mechanisms

Value Prediction

Prediction Error



Example Models

Bayesian Prediction & Evaluation

Behrens et al., 2007

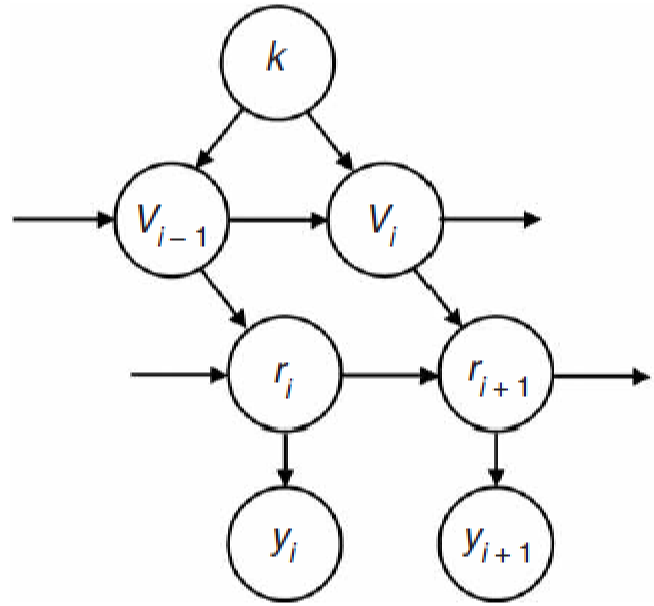
-Behavioral Output

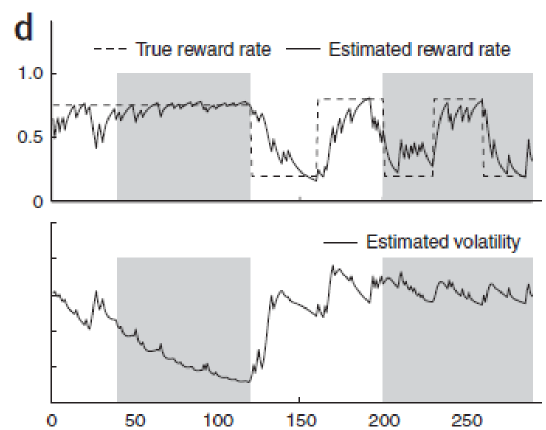
Choice

-Cognitive Mechanisms

Reward Probability

Environmental Volatility





Example Models

Drift Diffusion model

Ratcliff(1985)

-Behavioral output

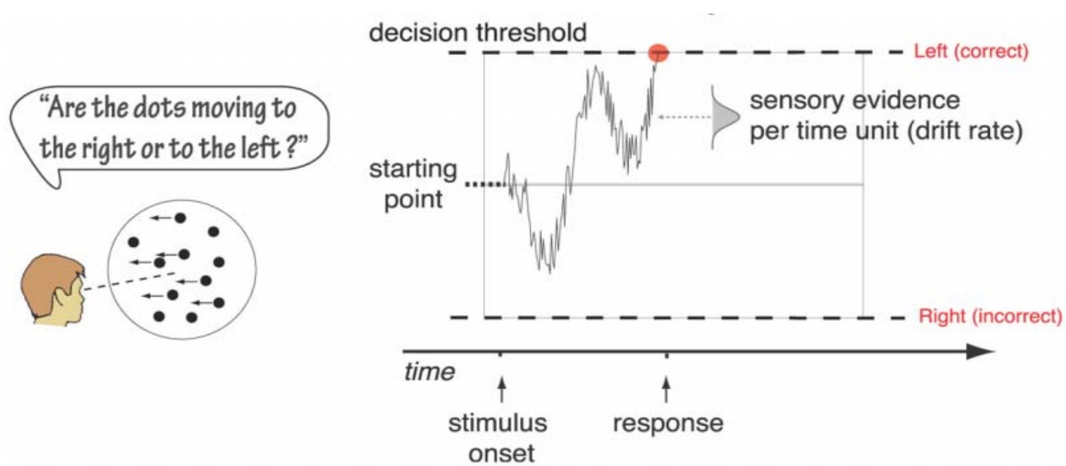
Choice

Reaction Time

-Cognitive Mechanisms

Information accumulation

Processing bias



Example Models

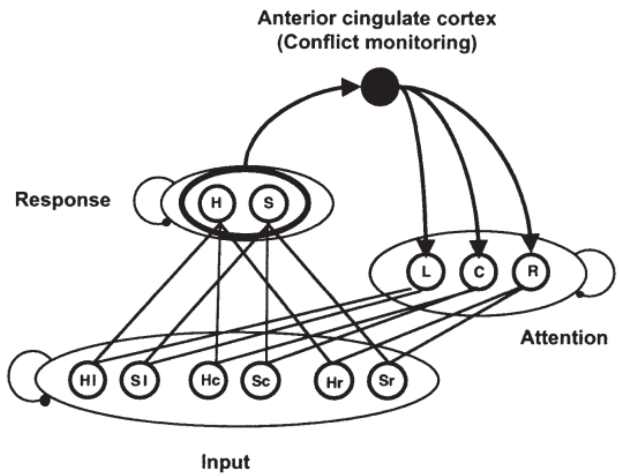
Conflict Model

Yeung et al. (2004)

-Behavioral output

Error Rate

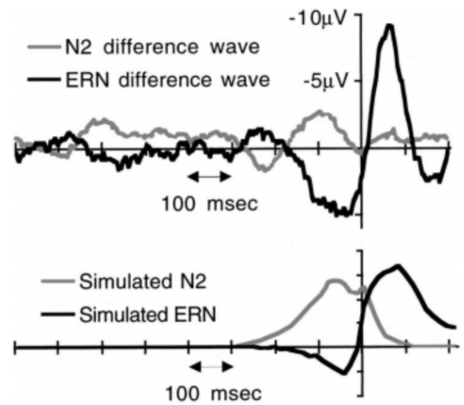
Reaction Time



-Cognitive Mechanisms

Behavioral Conflict

Attention



Example – Simple Reinforcement Learning

General Algorithm

•**Value Prediction** – Expected value of each option
•**Select Action** – Probabilistic choice of option with highest value
•**Comparison** – predicted outcome vs. actual outcome (prediction error)
•**Learning** – Update value predictions in proportion to discrepancy

Steps in Model-Based Analysis

1. Select/create a computational model
2. **Design experimental task in order to test model**
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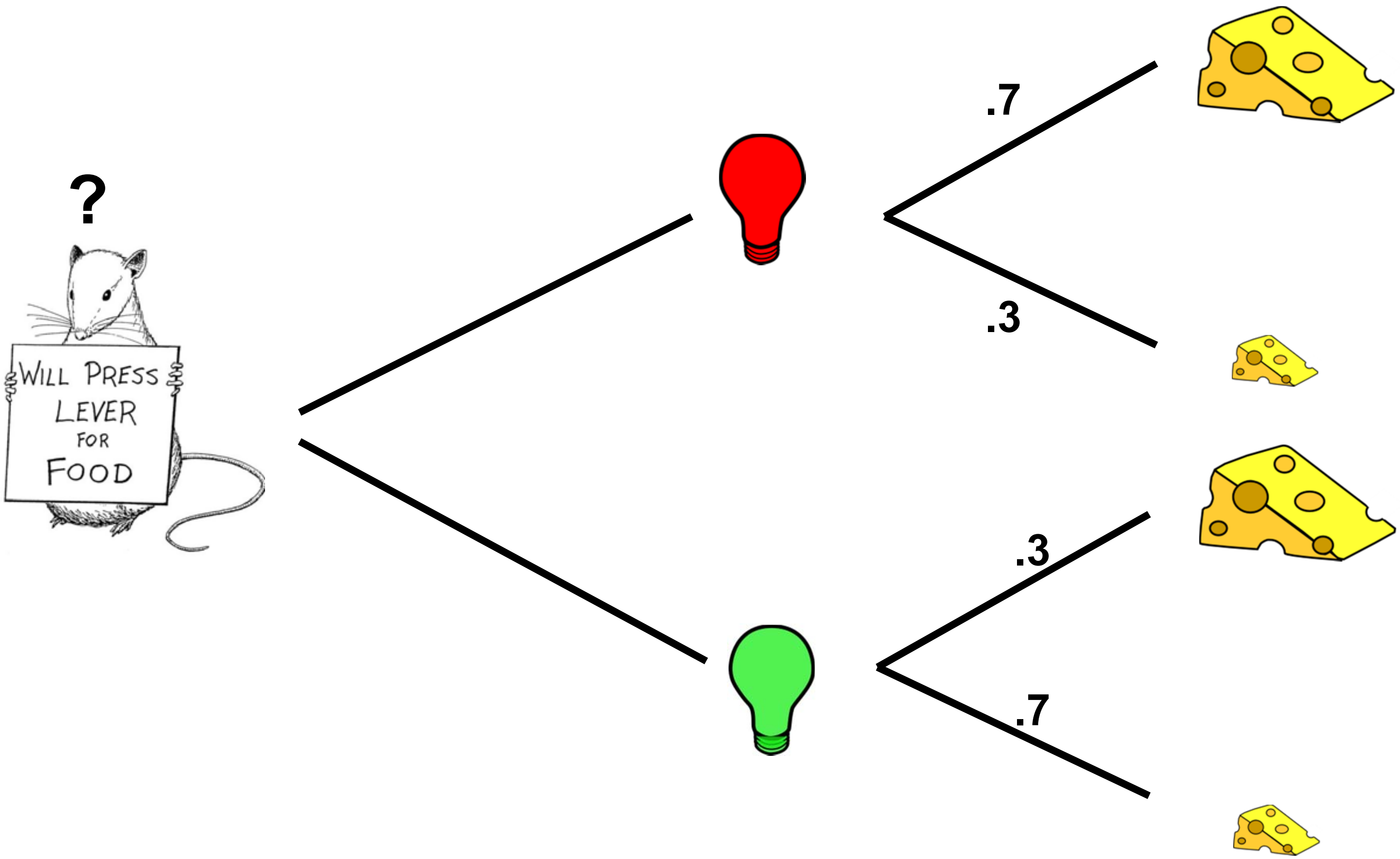
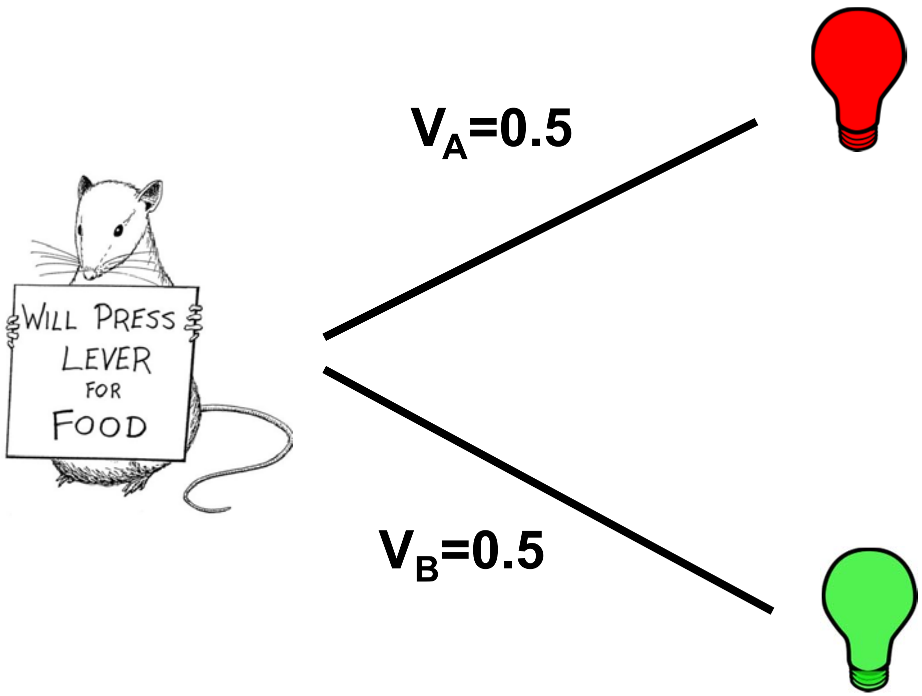
Example – Simple Reinforcement Learning



Example – Simple Reinforcement Learning

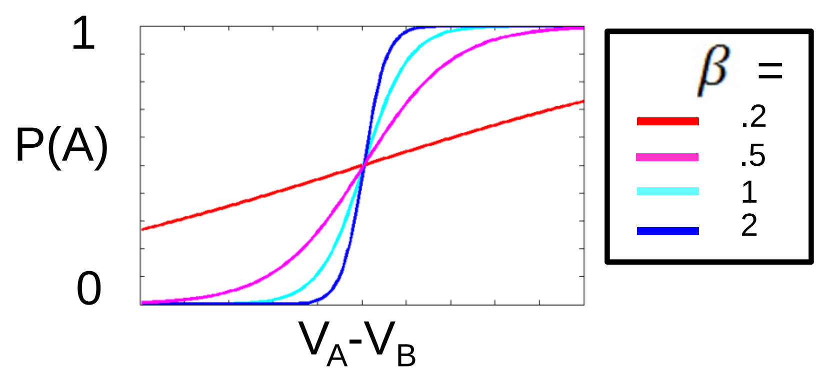
Value Prediction



Action Selection – Softmax:

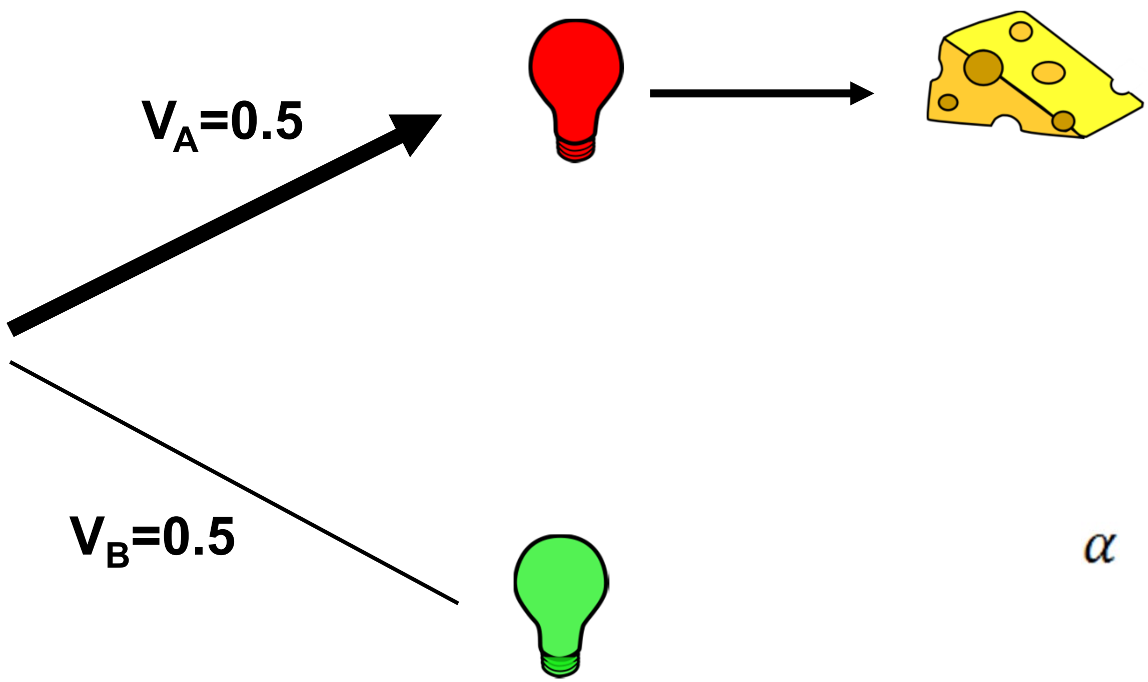
$$P(A) = \frac{exp(\beta * V_A)}{exp(\beta * V_A) + exp(\beta * V_B)}$$

$$P(B) = 1 - P(A)$$



β= inverse temperature parameter "decisiveness"

Example – Simple Reinforcement Learning



Comparison :

Outcome – Prediction

$$\delta = reward - V_A$$

Learning:

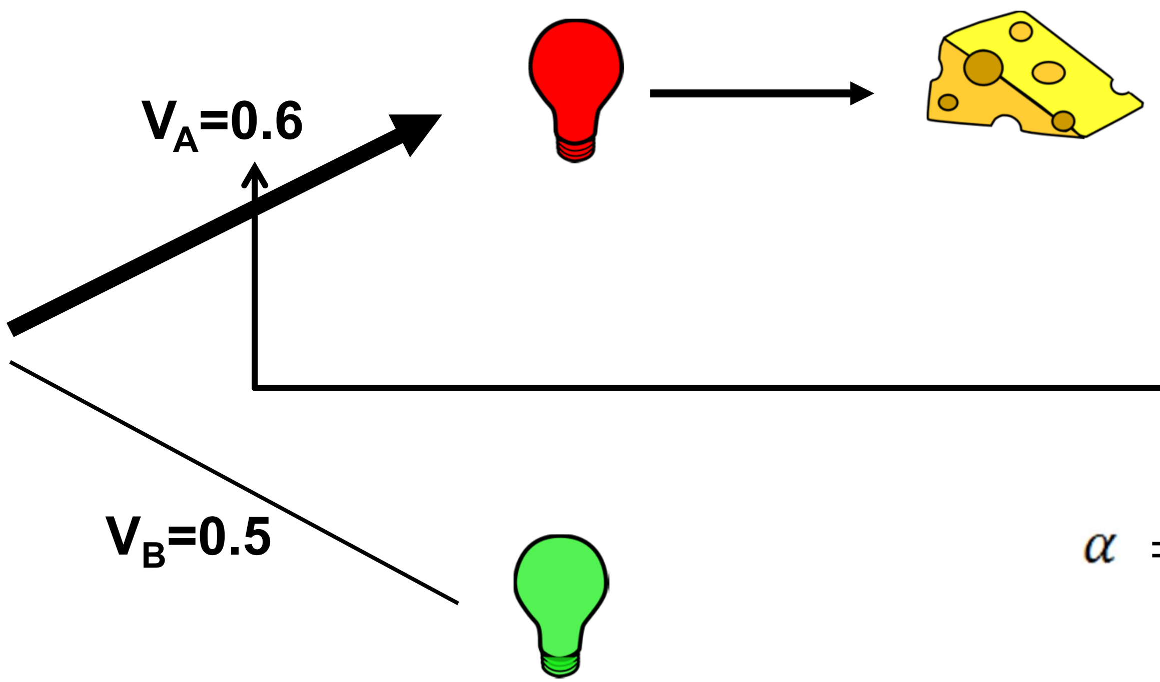
$$V_A = V_A + \alpha \times \delta$$

α = Learning Rate Parameter

Example – Simple Reinforcement Learning

Comparison :

Outcome – Prediction



$$\delta = reward - V_A$$

Learning:

$$V_A = V_A + \alpha \times \delta$$

α = Learning Rate Parameter

Model Predictions

Simple RL Model

-Behavioral output

Choice

-Cognitive Mechanisms

Value Prediction

Prediction Error

Governed by model parameters: α , β

Steps in Model-Based Analysis

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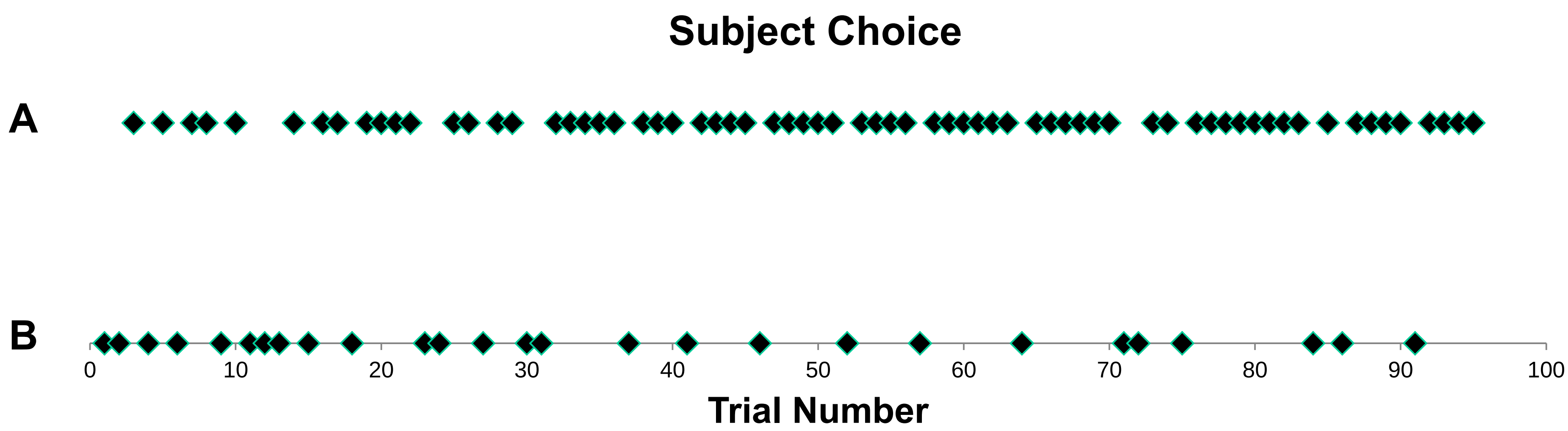
Fitting the model to data

Given our observed data and a model, what model parameters best explain the data?

Hypothetical Data



RL model Parameters

α Learning Rate

β Decision Temperature

Determining Parameter Values

Previously published parameter values
- may not apply to your data/task

"Reasonable" Parameters
- biologically/psychologically plausible
- open to interpretation

Optimization over a cost function
- comparison of the behavior of a model with subject behavior

Cost Functions

Ordinary Least Squares

- minimize sum of squared errors

$$\sum(\textit{Observed Behavior} - \textit{Model Prediction})^2$$

Maximum Likelihood Estimation

$$\ln \mathcal{L}(\theta \,|\, x_1, \ldots, x_n) = \sum_{i=1}^{n} \ln f(x_i|\theta),$$

Others . . .

Cost function minimization

Gradient descent on cost function

- Parameter values
- Calculate local gradient
- Adjust parameters
- Repeat until minimum reached

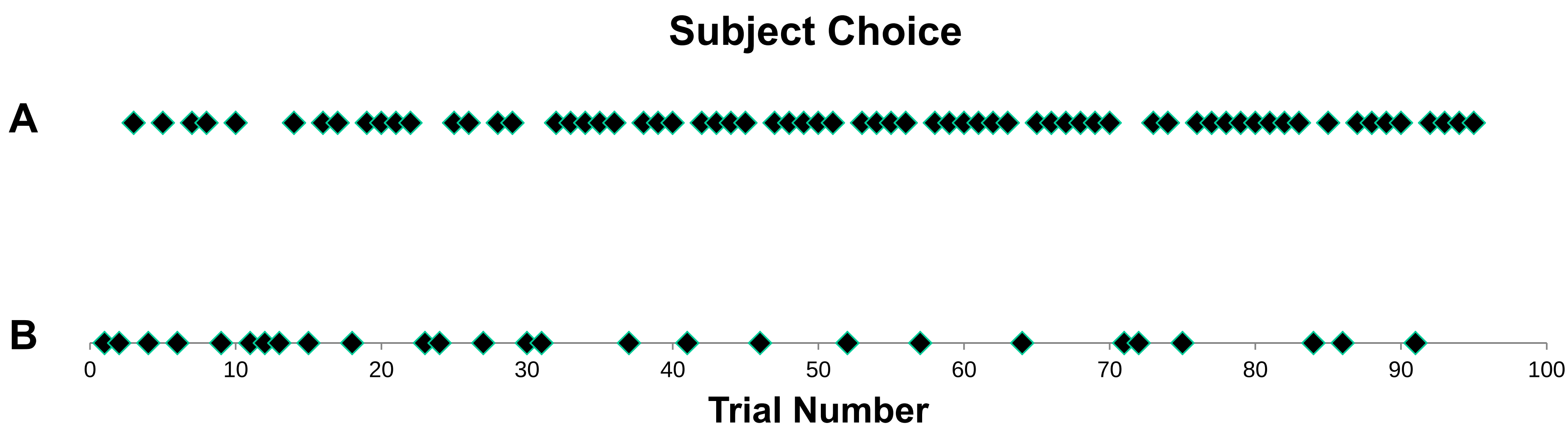
Existing tools

- e.g, MatLab fmincon/fminsearch
- Others...

Caveats

- Local minima vs. Global minimum
- Correlated parameters → unreliable estimates

Hypothetical Data



RL model Parameters

- α Learning Rate
- β Decision Temperature

Cost Function

$$\sum_t \Big(\underbrace{Choice_t}_{\textbf{Subject Behavior}} - \underbrace{P(V_t^{choice})}_{\textbf{Model Action Selection}}\Big)^2$$

Steps in Model-Based Analysis

1. Select/create a computational model
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Internal Model Variables

Simple RL model:

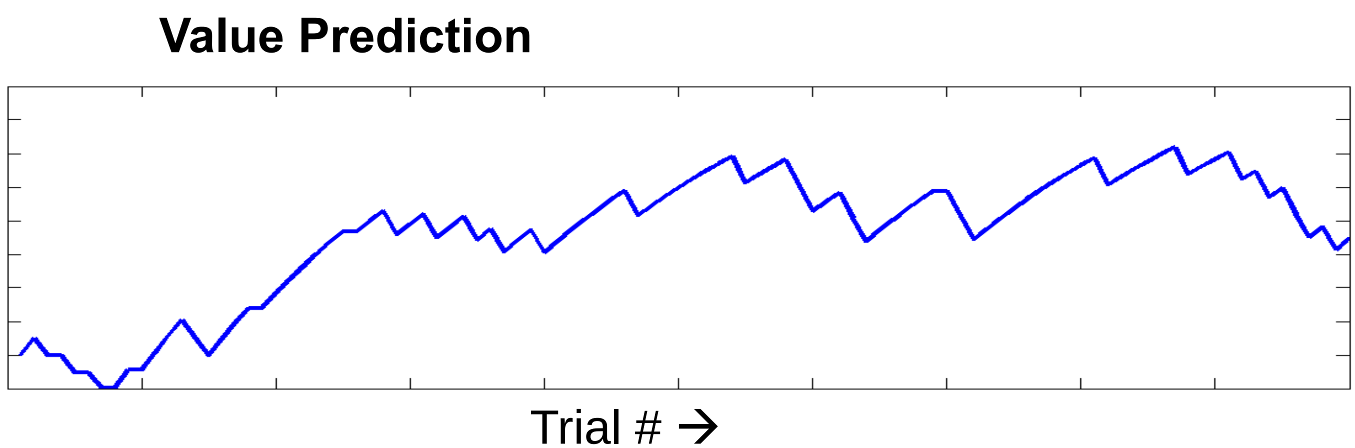
Estimated value of free parameters determines

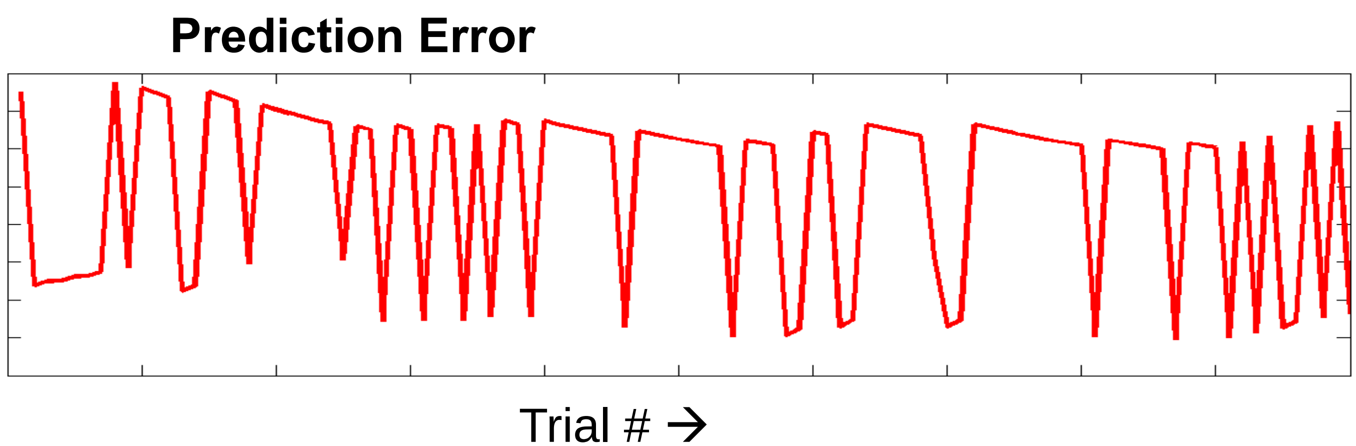
- How model learns/acts
- Development of internal model variables
  - Value Predictions
  - Prediction Errors

Internal model variables may be related to cognitive mechanism implemented by the brain

Model Variables → Brain Activity?

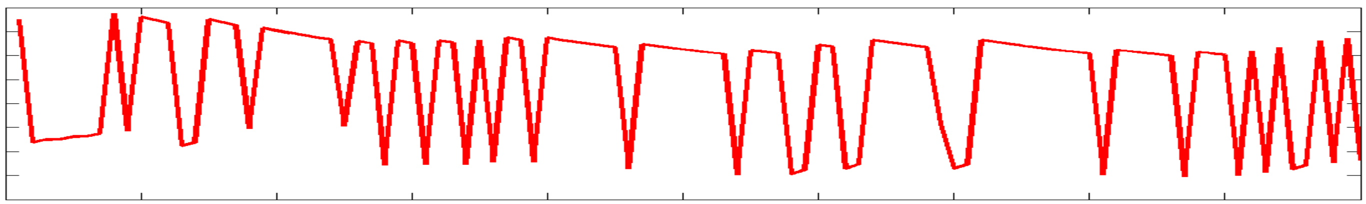
Internal Model Variables





Internal Model Variables

Prediction Error

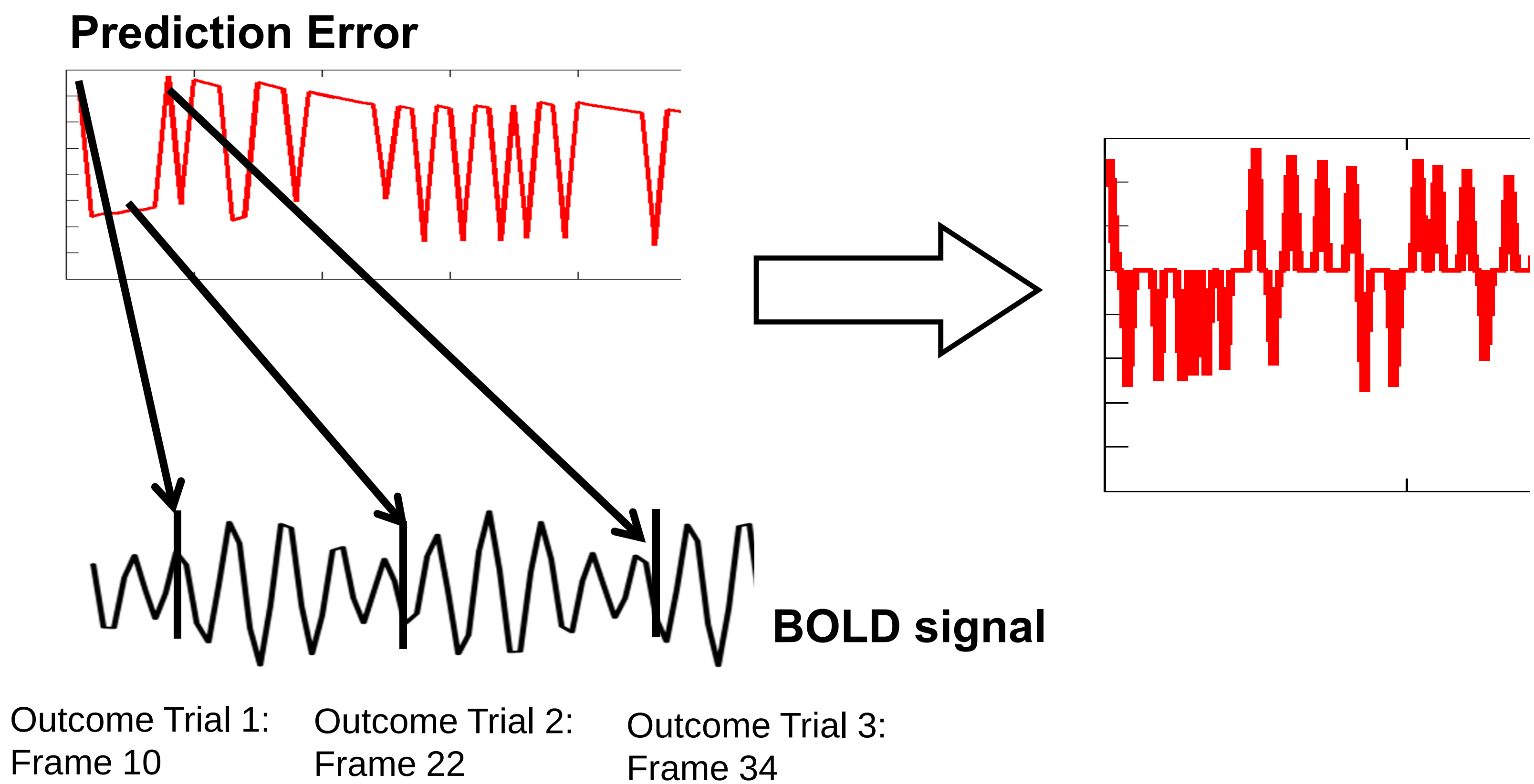


Trial # →

BOLD time course

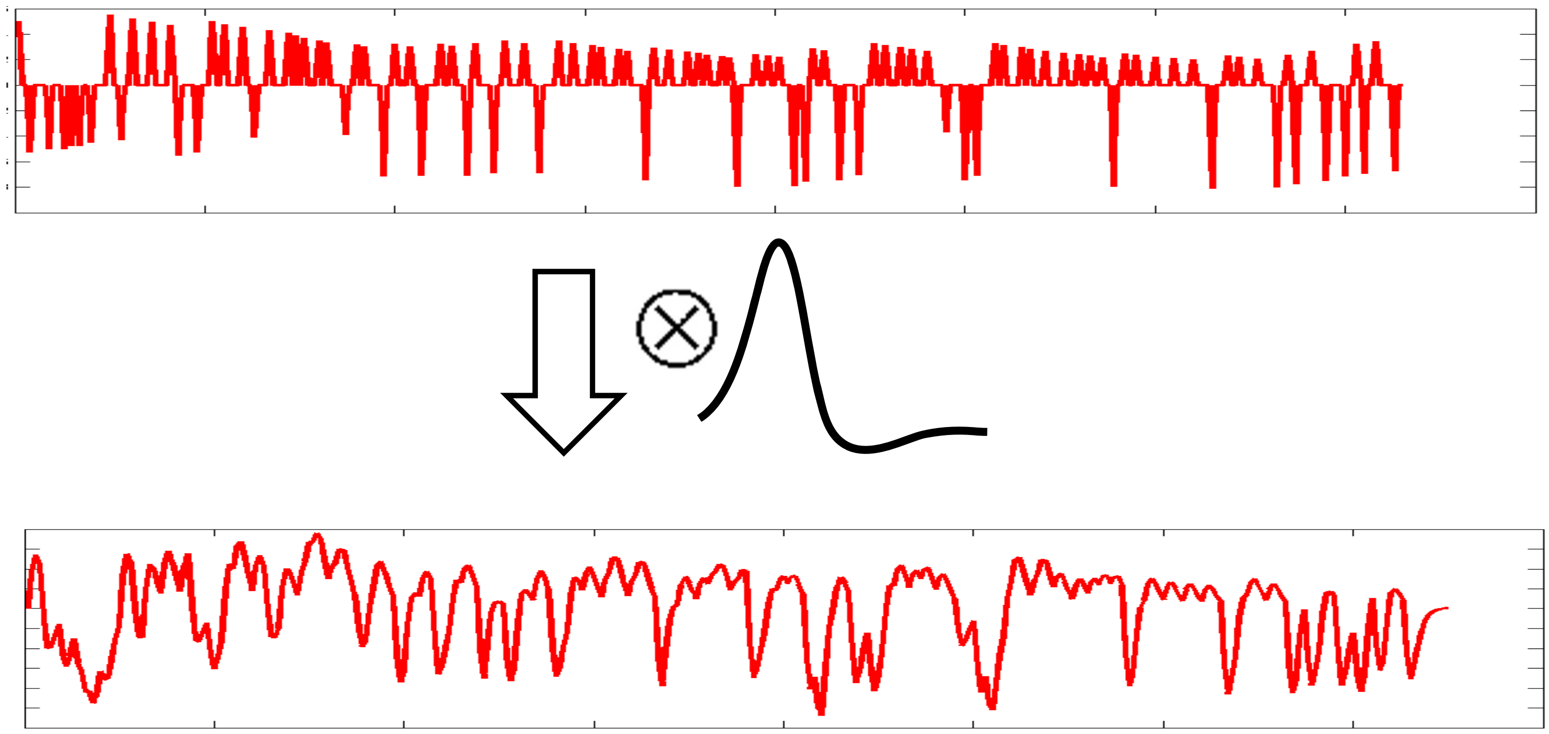
Model Variables to fMRI analysis

Resample model activity to match fMRI time course



Model Variables to fMRI analysis

Generate regressor from model variable:



Steps in Model-Based Analysis

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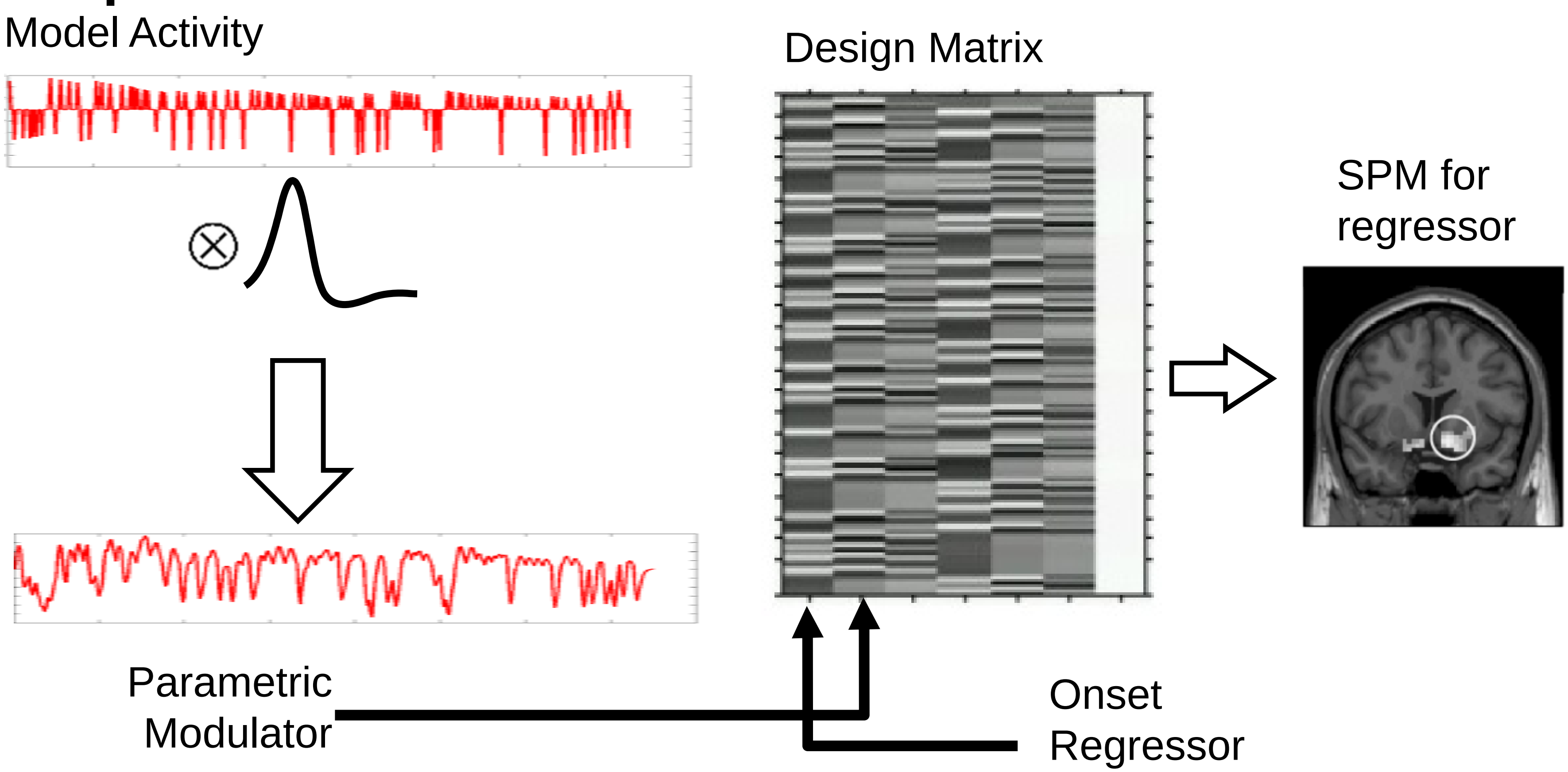
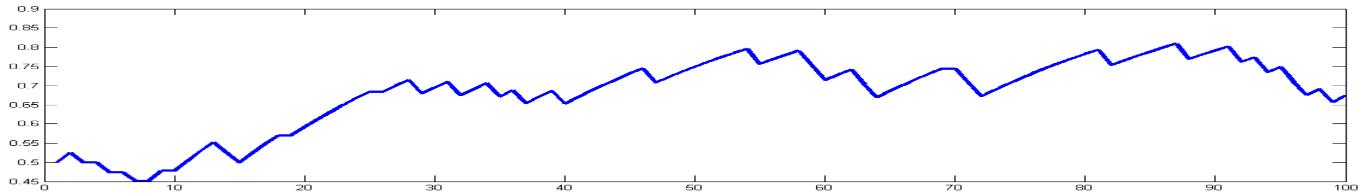
5. **Regress model predictions against fMRI data**

Add regressor to 1st level design matrix as parametric modulator



Internal Model Variables

Value Prediction



Trial # →

BOLD time course

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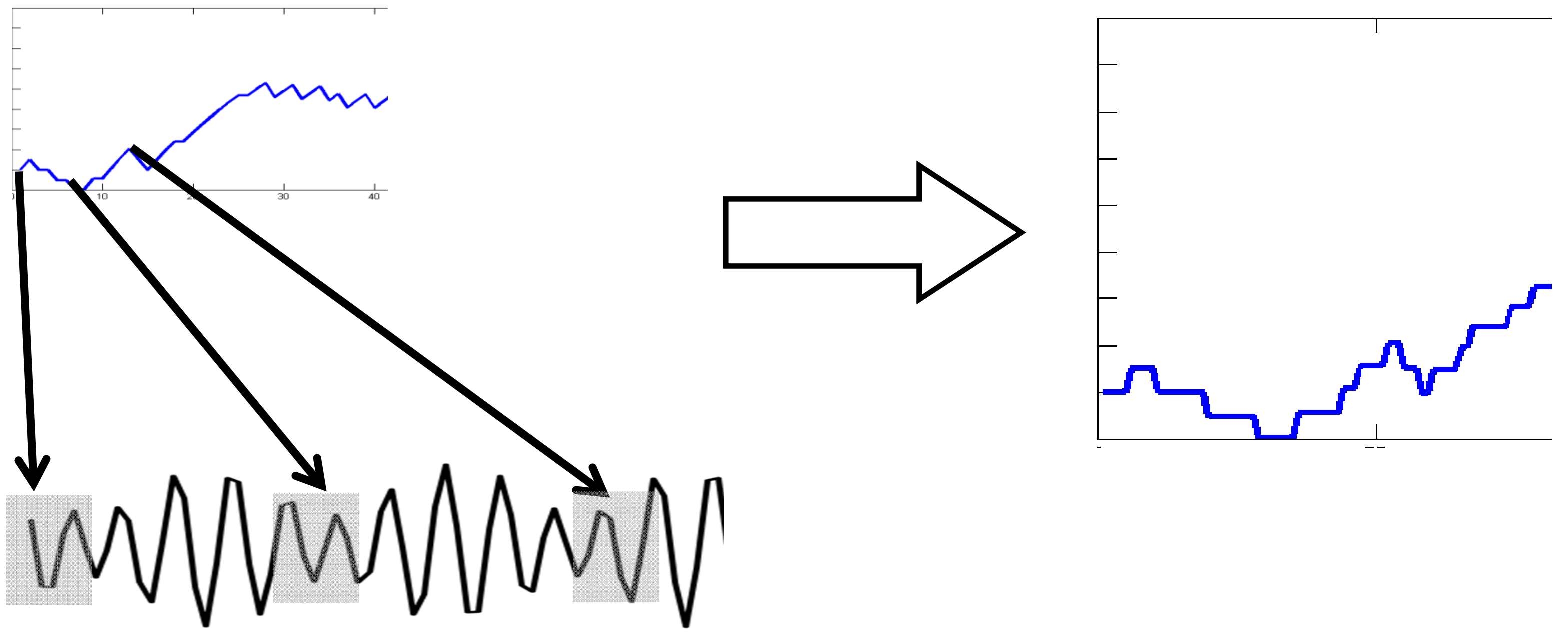
Model Variables to fMRI analysis

Resample model activity to match fMRI time course

Value Prediction



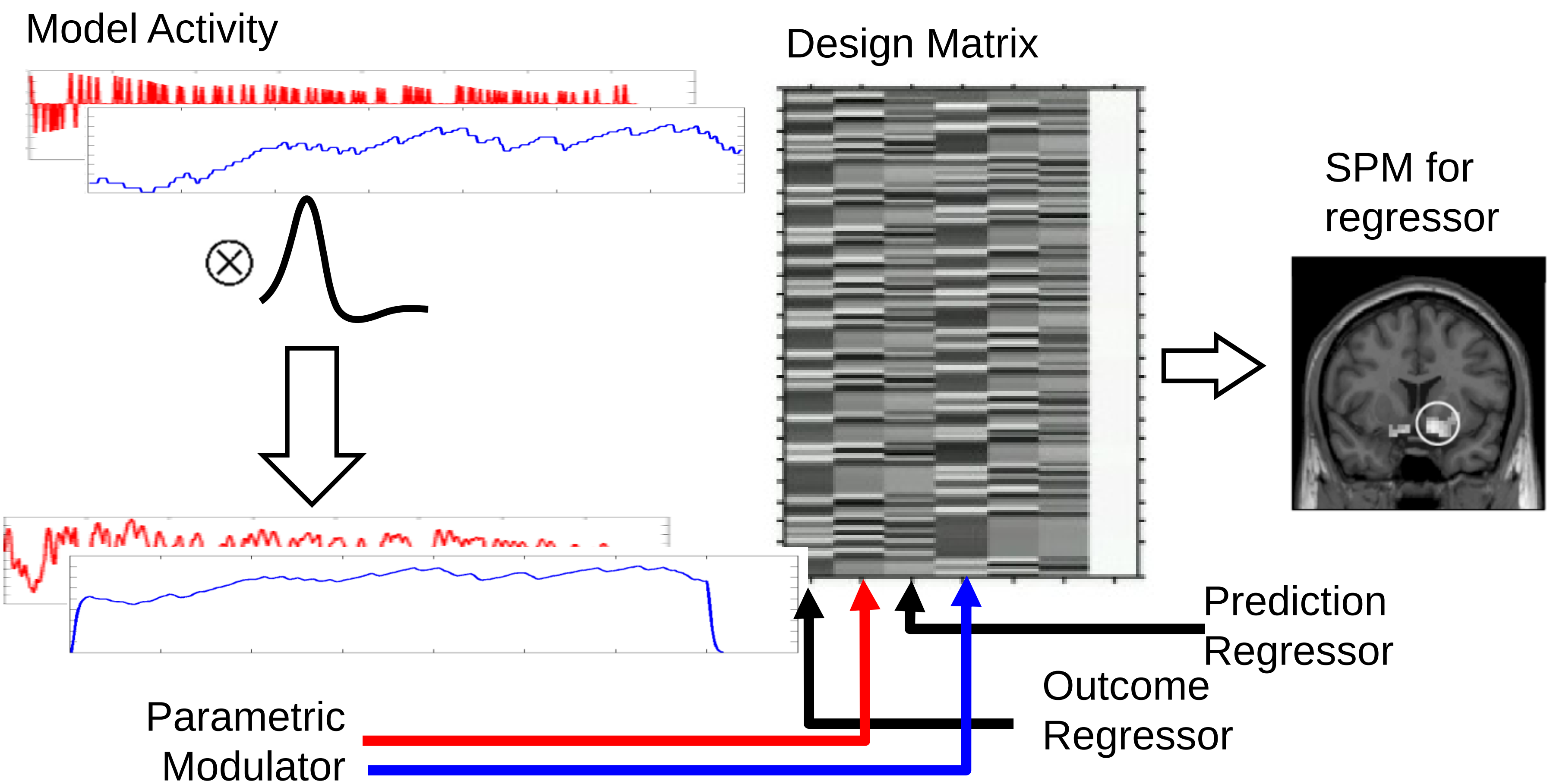
Prediction Trial 1: Frame 6-9

Prediction Trial 2: Frame 18-21

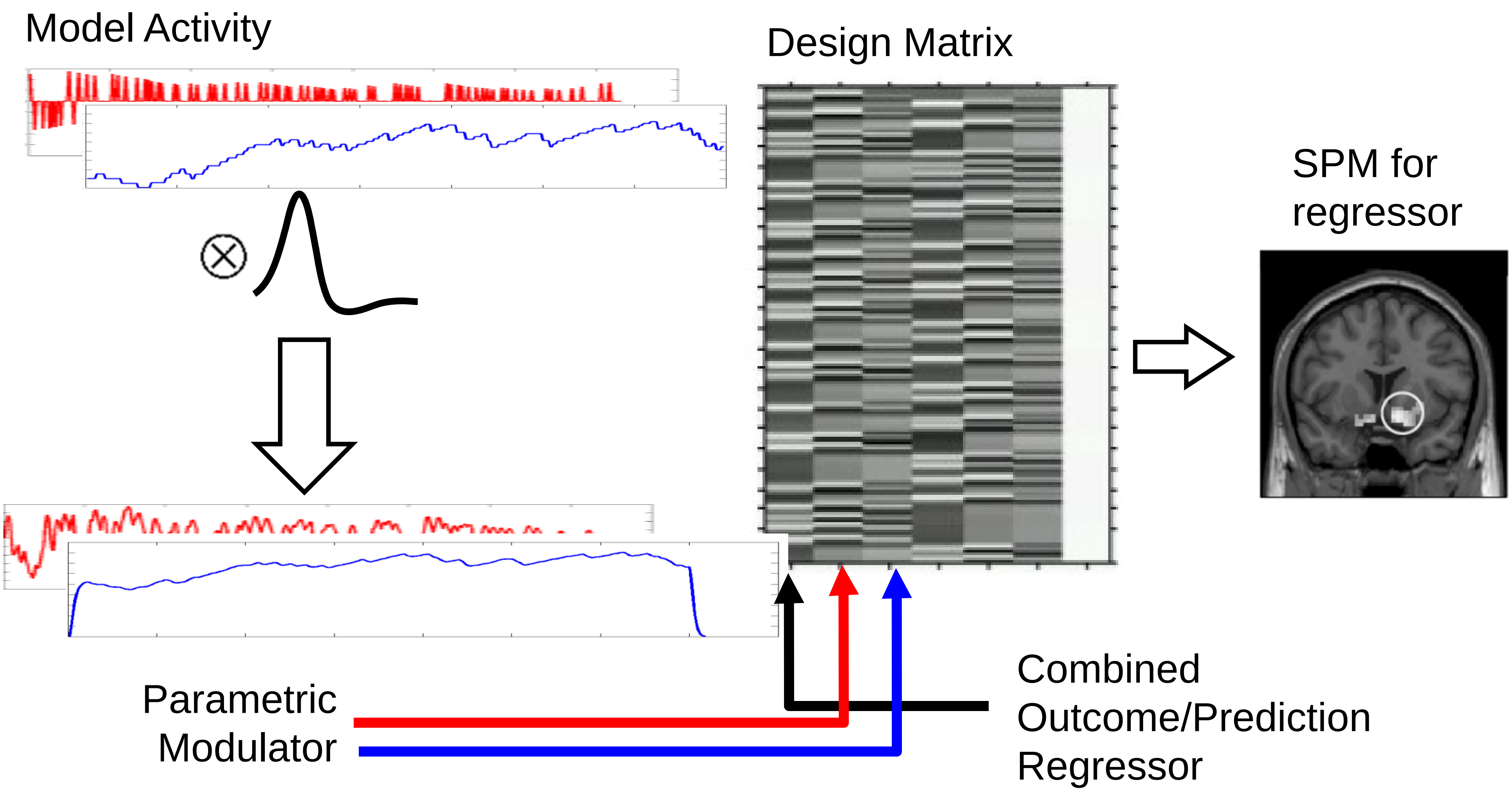
Prediction Trial 3: Frame 30-33

BOLD signal

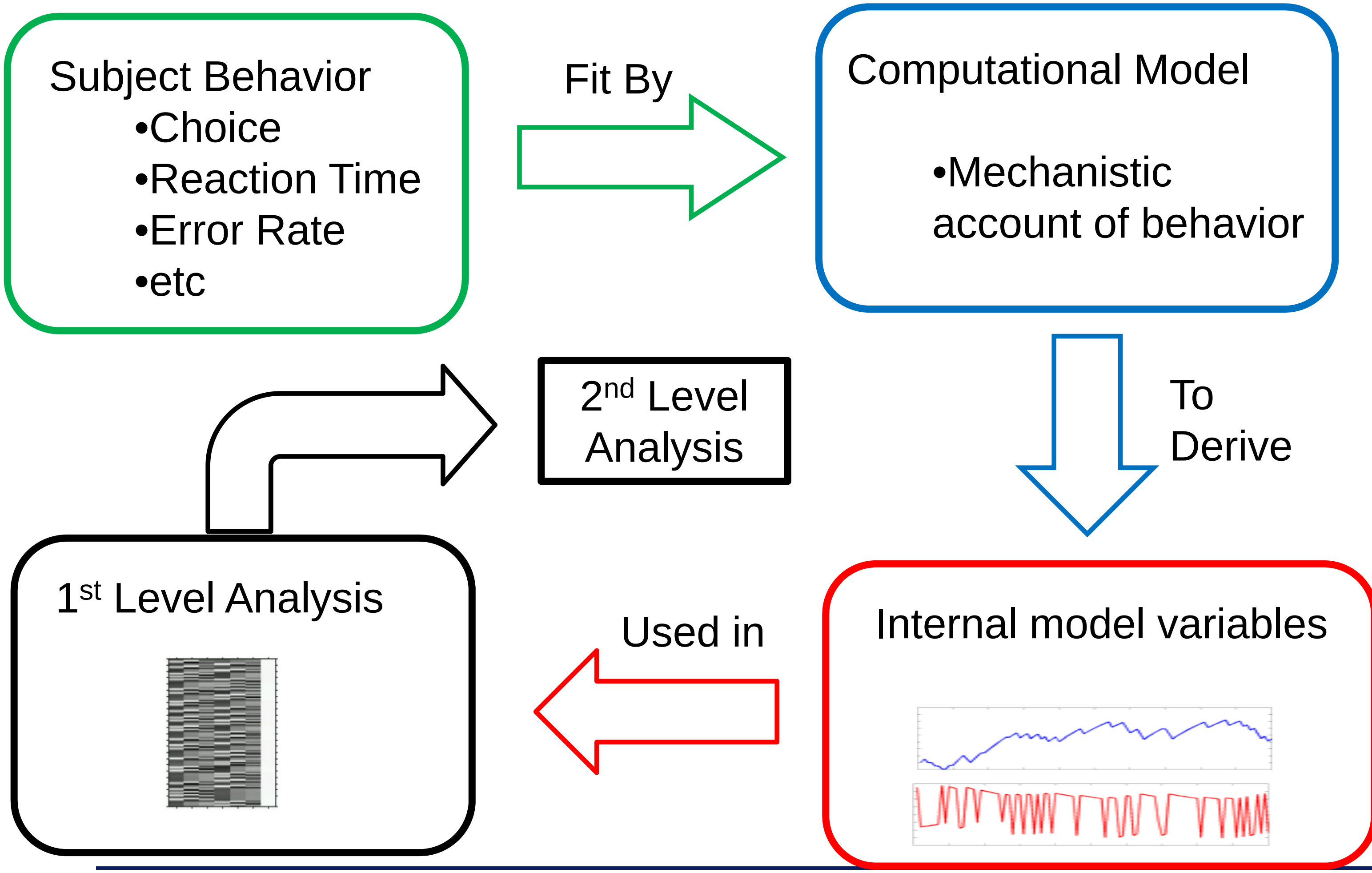
Multiple Model Regressors



Multiple parametric modulators



Model-Based fMRI analysis – Summary



Summary

You already use a model

Model-based fMRI analysis tries to determine how specific cognitive mechanisms may be implemented in the brain

Analysis is straightforward – most of the work is in the modeling

Many, many models to choose from

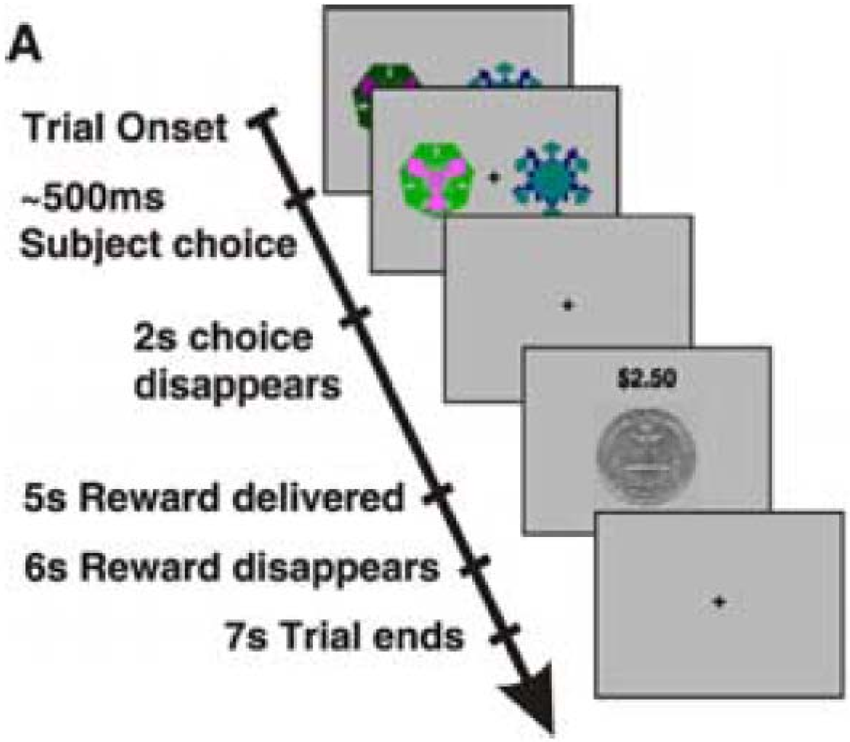
- Motivation
  - Models (general)
  - Why you ought to care
- Model-based fMRI
  - Models (specific)
  - From model to analysis
- Extended Example
  - Hampton, Bossaerts, & O’Doherty (2006)

8360 • The Journal of Neuroscience, August 9, 2006 • 26(32):8360–8367

Behavioral/Systems/Cognitive

The Role of the Ventromedial Prefrontal Cortex in Abstract State-Based Inference during Decision Making in Humans

Alan N. Hampton,[1] Peter Bossaerts,[1,2] and John P. O'Doherty[1,2]
[1]Computation and Neural Systems Program and [2]Division of Humanities and Social Sciences, California Institute of Technology, Pasadena, California 91125



Reward 70% of the time for correct choice, punished 30%
-Net Gain

Reward 40% for incorrect choice, punished 60%
-Net Loss

After choosing correct option 4 times consecutively, contingencies switched with probability .25

Model 1
Simple RL

Predictions:

$$V_a \qquad V_b$$

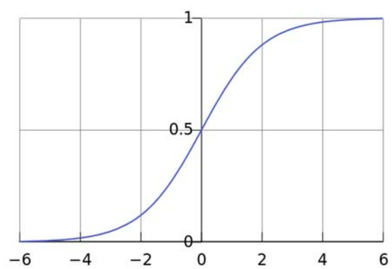
Learning:

$$V_a(t+1) = V_a(t) + \eta\delta(t),$$

$$\delta(t) = r(t) - V_a(t).$$

Choice:

$$P(A) = \sigma(\beta\{(V_a - V_b) - \alpha\})$$



Model 2
Hidden State Markov Model

Abstract Hidden State:

$X_t \rightarrow$ Correct/Incorrect choice

Infer probability of hidden state based on previous history, current choice:

$$P(X_t/X_{t-1}, S_t = stay) = \begin{pmatrix} 1-\delta & \delta \\ \delta & 1-\delta \end{pmatrix},$$

$$P(X_t/X_{t-1}, S_t = switch) = \begin{pmatrix} \delta & 1-\delta \\ 1-\delta & \delta \end{pmatrix},$$

Update estimates based on current trial:

$$\textbf{Posterior}(X_t = correct) = \frac{P(Y_t/X_t = correct)\textbf{Prior}(X_t = correct)}{\sum_{X_t\ states} P(Y_t/X_t)\textbf{Prior}(X_t)}$$

Probability of switch:

$$P(switch) = \sigma(\beta\{P_{incorrect} - \alpha\}).$$

Model Fitting

Model 1
Simple RL

$$P(A) = \sigma(\beta\{(V_a - V_b) - \alpha\})$$

Model 2
Hidden State Markov Model

$$P(switch) = \sigma(\beta\{P_{incorrect} - \alpha\}).$$

$$\log L = \frac{\sum B_{switch} \log P_{switch}}{N_{switch}} + \frac{\sum B_{stay} \log P_{stay}}{N_{stay}}$$

Behavioral Data

Model Parameters

Model 1
Simple RL

$$P(A) = \sigma(\beta\{(V_a - V_b) - \alpha\})$$

$$P(switch) = \sigma(\beta\{P_{incorrect} - \alpha\}).$$

Temperature – exploration/exploitation

Indifference point

$$V_a(t+1) = V_a(t) + \eta\delta(t),$$

Learning Rate

Model 2
Hidden State Markov Model

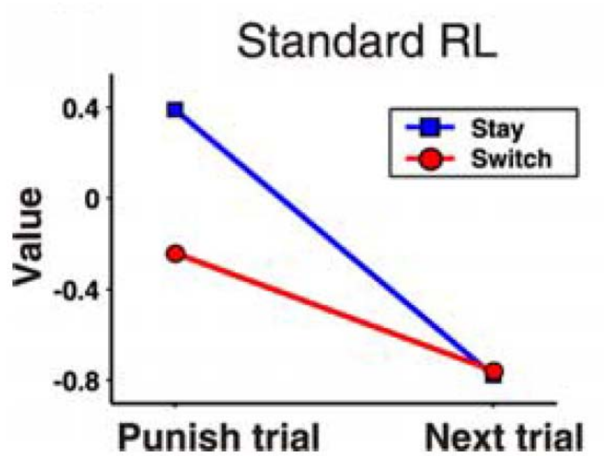
$$P(X_t/X_{t-1}, S_t = stay) = \begin{pmatrix} 1-\delta & \delta \\ \delta & 1-\delta \end{pmatrix},$$

$$P(X_t/X_{t-1}, S_t = switch) = \begin{pmatrix} \delta & 1-\delta \\ 1-\delta & \delta \end{pmatrix}$$

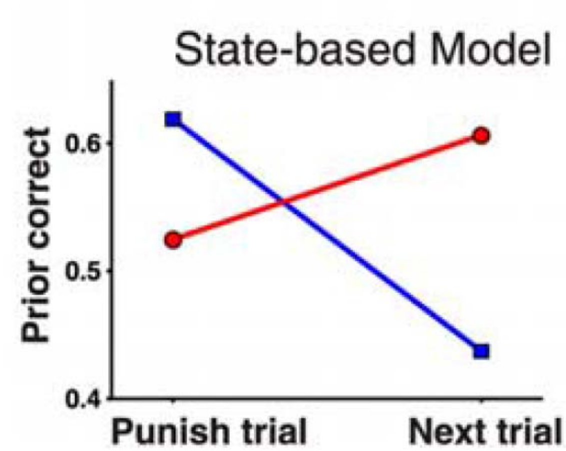
Probability of reversal

Model Predictions

Model 1
Simple RL



Model 2
Hidden State Markov Model

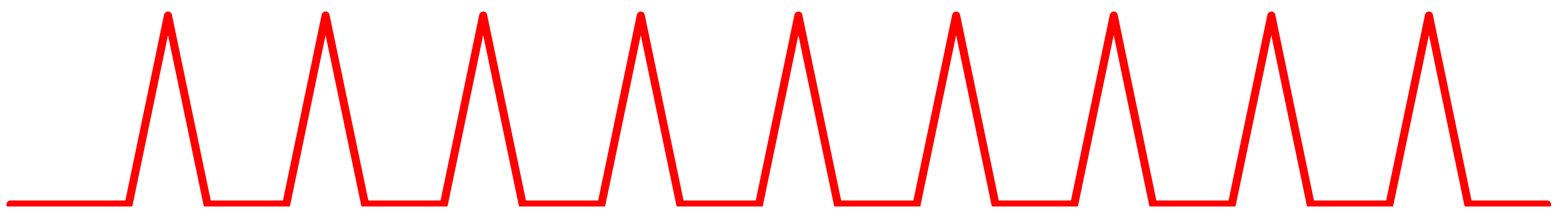


fMRI Analysis

Regressors:

Delta function at choice

and outcome



fMRI Analysis

Parametric Modulation of Choice

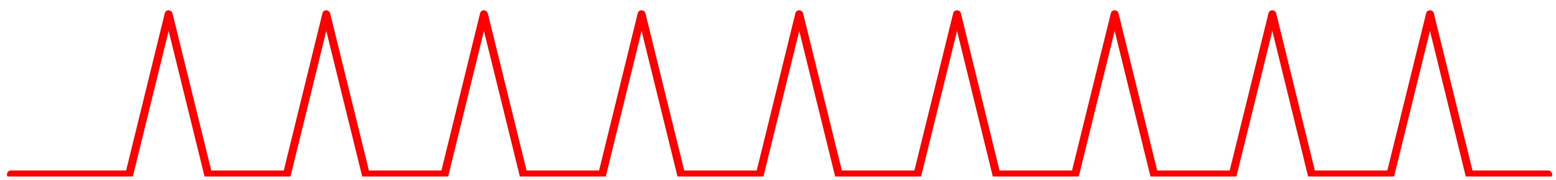
Delta function at Choice

RL Model - Value Prediction **HMM – P(X=correct)**

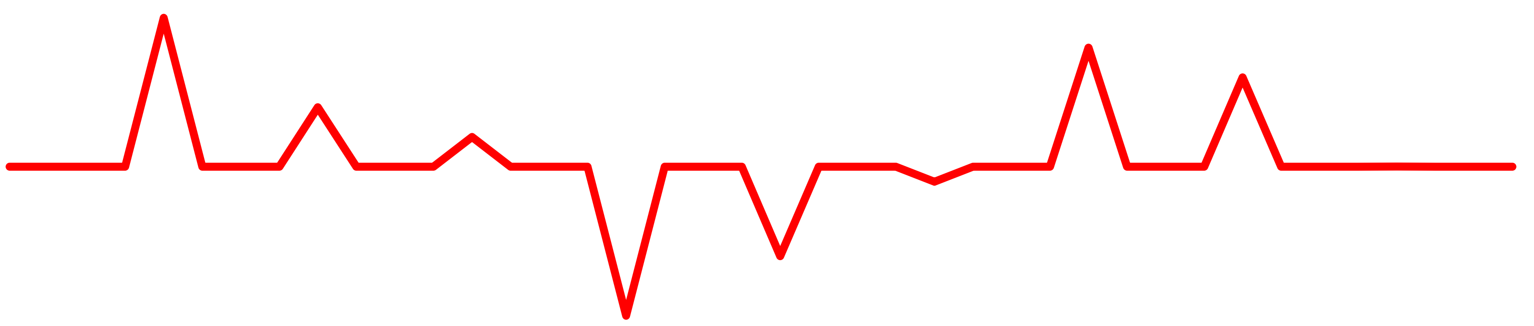
fMRI Analysis

Parametric Modulation of Outcome

Delta function at Outcome

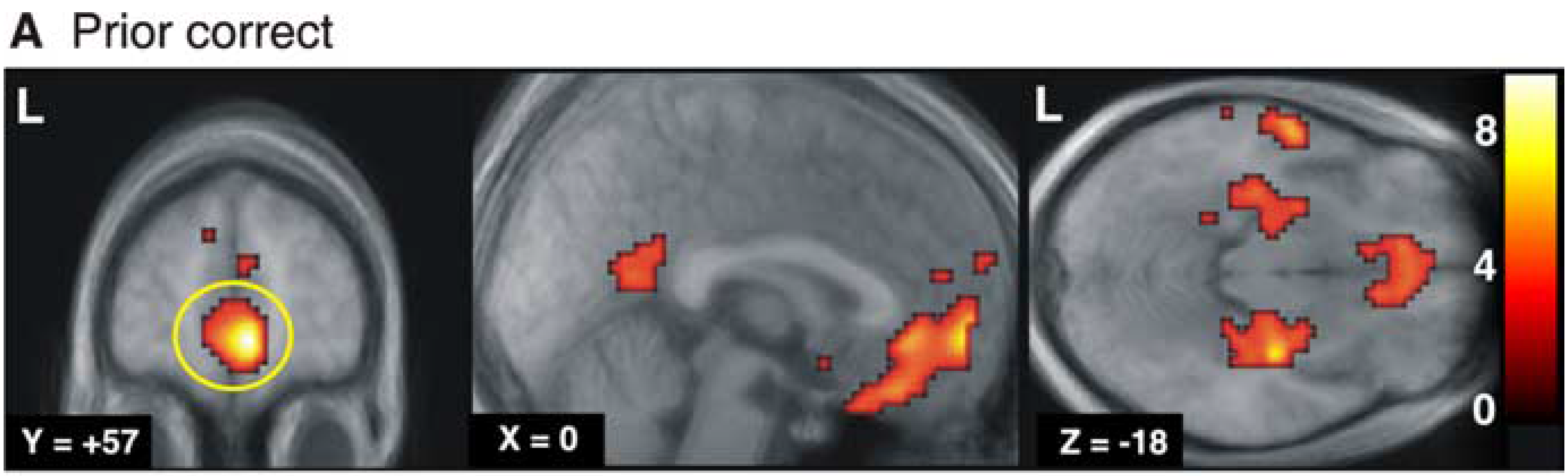


RL Model – Prediction Error **HMM – (Posterior Probability – Prior Probability)**



fMRI Results

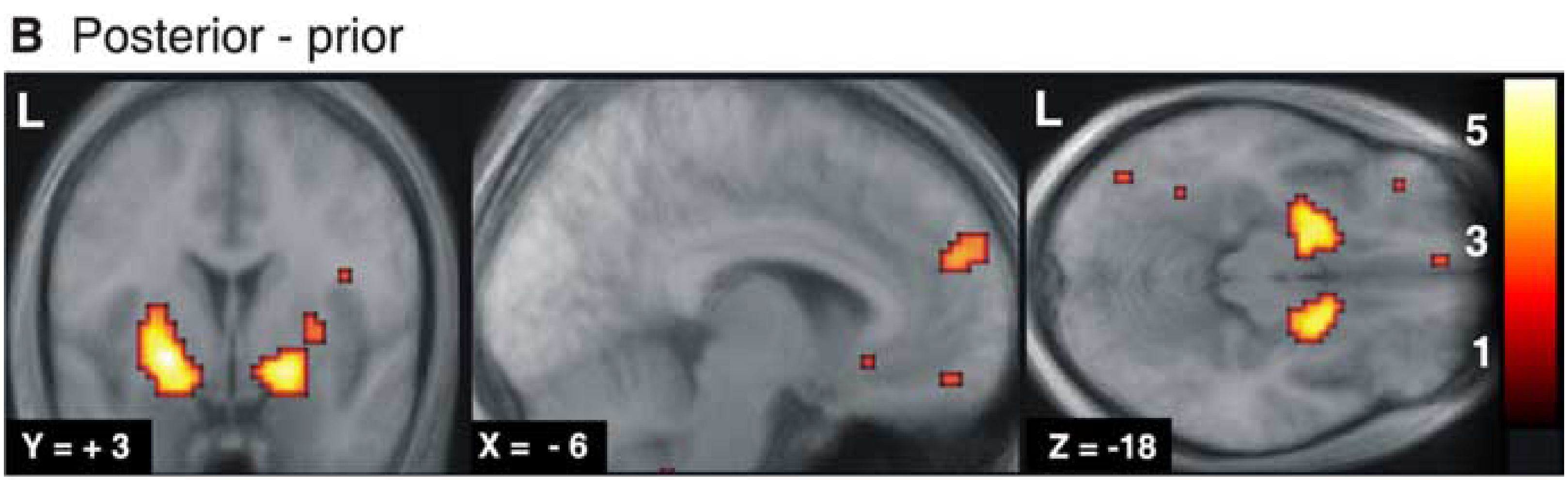
HMM Model (Choice): P(X=correct)



Time locked to time of choice

fMRI Results

HMM Model (Outcome): Posterior – Prior



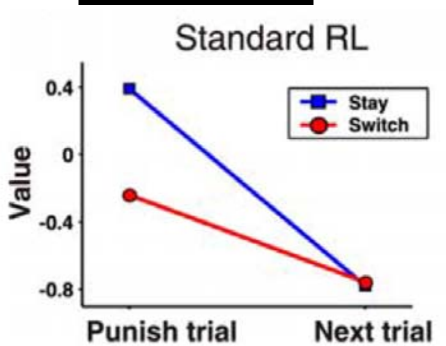
fMRI Results

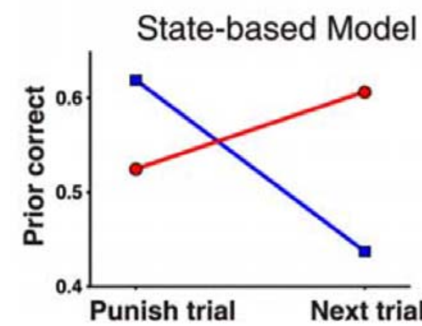
Comparison of Model Predictions

Model 1
Simple RL

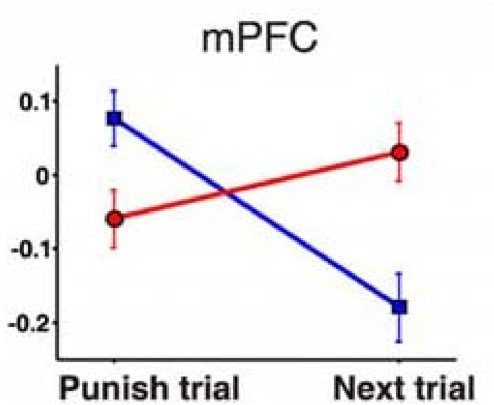
Model 2
Hidden State Markov Model

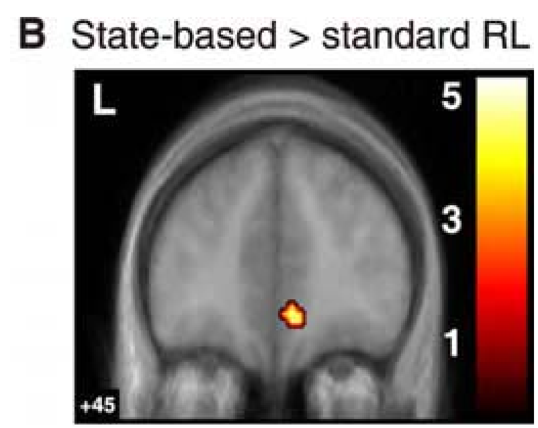
Predictions





Results





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